Estimation of the seismic safety for multistory steel structures by a criterion based on energy

E. Tulei, D. Cretu, Department of Strength of Materials, Technical University of Civil Engineering, Bucharest

Abstract: Steel structure capacity of yielding under severe earthquakes permits a significant reduction of the seismic forces evaluated for an ideal linear elastic behaviour. Estimation of the *q*-reduction factor represents a continuous preoccupation, sustained by many studies in the last decades. Different approaches of this problem are possible, based on pushover or dynamic methods.

This paper presents the primary theoretical elements for the evaluation of the behaviour factor *q* by a global criterion based on energy. The method is illustrated by the case study of a 15-story eccentrically braced steel frame subjected to 1977, 1986 and 1990 INCERC-Bucharest earthquakes.

Nonlinear dynamic analysis for a great number of accelerograms is the only method to estimate more realistically the ductility demand, as an expression of the seismic structural performance.

1. Introduction

The simplified structural design performed by utilizing reduced seismic forces, relies on the control of the plastic hinge development. The expected behaviour of the structure must be verified through nonlinear static or dynamic analysis.

The pushover analysis may indicate an overstrength (ultimate/design base shear) of the structure due, for instance, to the dimensions of the cross-sections, chosen often bigger than it is necessary.

The time-history analysis provides the locations of the plastic hinges under the severest action. The time-dependent response must be obtained for a number of accelerograms. Subsequently, the dimensioning forces are selected. So far, recorded accelerograms are very few and only for several sites, which makes uncertain the correct modelling of the seismic action. On the other hand, frame members usually are subjected to combined forces. Therefore, the choice of all possible force combinations at the same moment is necessary, which increases the computational effort.

A nonlinear static or dynamic analysis must consider the following aspects: the control of plastic hinges development and global collapse mechanism formation; the estimation of the ductility demand, which must be compared with the global structure ductility that was considered at the reduction of the seismic forces; the displacement control, in order to avoid second order effects; the drift control, as a measure of the ductility demand; the control of the plastic potential section rotations, to ensure and verify the rotational ductility of the dissipative members; appreciation of the structural damping in dissipating the earthquake input energy; estimation, from the energetic principles, of the structure capacity to dissipate energy by inelastic deformations.

2. Nonlinear dynamic response analysis

The response to seismic actions is obtained by time-stepping integration of the equations of motion, considering the inelastic behaviour of the structural elements. Yielding occurs in a section when the limit capacity is reached. If the state of the section modifies during a time step, the equilibrium equations are not satisfied in the new state, at the end of the time step. In this case, the computation errors must be identified, and the equilibrium can be recovered using correctional forces applied in the next time step, or successive corrections during the time step. Choosing a smaller time step reduces the errors.

Nonlinear dynamic response is influenced by: the time step size; the variation law accepted for the relative accelerations over the time step; the inertial properties idealization (lumping the masses at the structure nodes and identifying the number of the dynamic degrees of freedom); structural damping idealization; dead load idealization; the considering of the axial force effect.

From among the available integration methods of the equations of motion, the average acceleration method is simple and leads to acceptable results for practical goals. The time step is usually taken to be constant. The matrices from the incremental dynamic equilibrium equations are evaluated at the beginning of each time step. The response at the end of the time step is obtained accepting that coefficients $\mathbf{c}(\dot{\mathbf{u}})$ and $\mathbf{k}(\mathbf{u})$ of the damping and stiffness matrices, respectively, remain constant during the time step. The nonlinearity is taken into account by re-evaluating these coefficients at the beginning of the next time step. Possible behaviour modifications are imposed by the global structural response and can be evaluated only at the end of the time step. The transition from an elastic state to a plastic state or conversely is identified by the position of the point having as coordinates the end section forces of the elements with respect to the limit interaction curve or surface. The displacement and velocity calculated at the end of a time step become initial conditions for the next time step. Consequently, the nonlinear behaviour of a structural system can be approximated by a succession of linear systems that change with every time step.

The accuracy of the solution depends on the time step size, that has to be chosen according to the natural frequencies of vibration of the structure, the time variation of the dynamic excitation, the complexity of the functions defining damping and stiffness.

The time step should be sufficiently small to permit a close approximation of the ground acceleration and to detect the response peaks and the transitions at sharp corners in the force-deformation curves of elastoplastic systems.

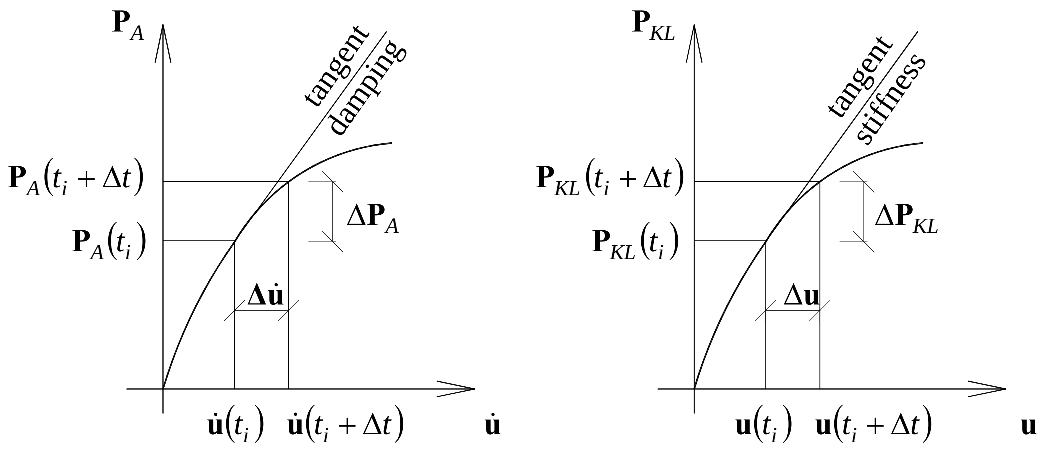
The incremental equation of motion to be solved is

$$\mathbf{M}\Delta\ddot{\mathbf{u}} + \mathbf{C}_t\Delta\dot{\mathbf{u}} + \mathbf{K}_t\Delta\mathbf{u} = -\mathbf{M}\,\mathbf{1}\,\Delta\ddot{u}_g$$

or (1)

$$\Delta\mathbf{P}_M + \Delta\mathbf{P}_A + \Delta\mathbf{P}_{KL} = \Delta\mathbf{P}_g$$

where $\mathbf{u}$, $\dot{\mathbf{u}}$ and $\ddot{\mathbf{u}}$ are the structure relative displacement, velocity and acceleration vectors, $\ddot{u}_g$ is the ground acceleration, and $\mathbf{M}$ is the mass matrix, $\mathbf{C}_t$ is the tangent damping matrix and $\mathbf{K}_t$ is the tangent stiffness matrix. $\mathbf{C}_t$ and $\mathbf{K}_t$ are computed at the beginning of the time step Δt, and considered constant during the time step (Fig. 1).



$$\mathbf{C}_t = \left(\frac{\mathbf{dP}_A}{\mathbf{d\dot{u}}}\right)_{t_i} ; \; \mathbf{K}_t = \left(\frac{\mathbf{dP}_{KL}}{\mathbf{du}}\right)_{t_i}$$

Figure 1 Tangent damping and tangent stiffness matrices

The steps to be followed in order to solve the matrix differential equations (1) are:

– selection of the relative acceleration variation over the time step Δt;

– expression of $\Delta\dot{\mathbf{u}}_i$ and $\Delta\ddot{\mathbf{u}}_i$ as functions of the displacement variation over the time step, $\Delta\mathbf{u}_i$, and of the velocities $\dot{\mathbf{u}}_i$ and accelerations $\ddot{\mathbf{u}}_i$ from the beginning of the time step;

– substitution of $\Delta\dot{\mathbf{u}}_i$ and $\Delta\ddot{\mathbf{u}}_i$ into Eq. (1). The following equation is obtained:

$$\overline{\mathbf{K}}_i\,\Delta\mathbf{u}_i = \overline{\Delta\mathbf{P}}_i$$

where

$$\overline{\mathbf{K}}_i = \mathbf{K}_i + a_1\mathbf{M} + a_2\mathbf{C}_i$$

and

$$\overline{\mathbf{\Delta P}}_i = \mathbf{\Delta P}_{g,i} + (a_3\dot{\mathbf{u}}_i + a_4\ddot{\mathbf{u}}_i)\mathbf{M} + (a_5\dot{\mathbf{u}}_i + a_6\ddot{\mathbf{u}}_i)\mathbf{C_i}$$

$a_1 \div a_6$ are:

$$a_1 = \frac{1}{\beta(\Delta t)^2};\ a_2 = \frac{\gamma}{\beta\,\Delta t};\ a_3 = \frac{1}{\beta\,\Delta t}$$

$$a_4 = \frac{\gamma}{\beta};\ a_5 = \frac{1}{2\beta};\ a_6 = \left(\frac{\gamma}{2\beta} - 1\right)\Delta t$$

The parameters β and γ define the variation of the acceleration over the time step, Δt. For linear variation, $\gamma = \frac{1}{2}$ and $\beta = \frac{1}{6}$, while for constant average acceleration, $\gamma = \frac{1}{2}$ and $\beta = \frac{1}{4}$. The linear acceleration method and the average acceleration method are special cases of the Newmark's time-stepping method.

– solving the linear system of equations

$$\Delta\mathbf{u}_i = \overline{\mathbf{K}}_i^{-1}\mathbf{\Delta P}_i$$

– calculation of $\mathbf{\Delta}\dot{\mathbf{u}}_i$ and $\mathbf{\Delta}\ddot{\mathbf{u}}_i$ from the known $\mathbf{\Delta u}_i$, $\dot{\mathbf{u}}_i$ and $\ddot{\mathbf{u}}_i$;

– calculation of displacements and velocities at the end of the time step:

$$\mathbf{u}_{i+1} = \mathbf{u}_i + \mathbf{\Delta u}_i,\ \dot{\mathbf{u}}_{i+1} = \dot{\mathbf{u}}_i + \mathbf{\Delta}\dot{\mathbf{u}}_i$$

– calculation of accelerations $\ddot{\mathbf{u}}_{i+1}$ at the end of the time step, from the equilibrium condition:

$$\ddot{\mathbf{u}}_{i+1} = \mathbf{M}^{-1}(\mathbf{P}_{i+1} - \mathbf{C}_{i+1}\dot{\mathbf{u}}_{i+1} - \mathbf{K}_{i+1}\mathbf{u}_{i+1})$$

The stiffness matrix at the end of the time step, $\mathbf{K}_{i+1}$, is the same from the beginning of the time step, or is modified if the structure developed inelastic deformations. The behaviour range is controlled at the end of every time step.

– calculation of end section forces at the end of the time step, $\mathbf{F}_{i+1} = \mathbf{F}_i + \mathbf{\Delta F}_i$.

– comparison of the force level with the accepted limit interaction curve. The behaviour state of the section may be linear elastic, inelastic through deformation in one sense (loading) and inelastic through deformation in reverse sense (unloading) (Fig. 2).

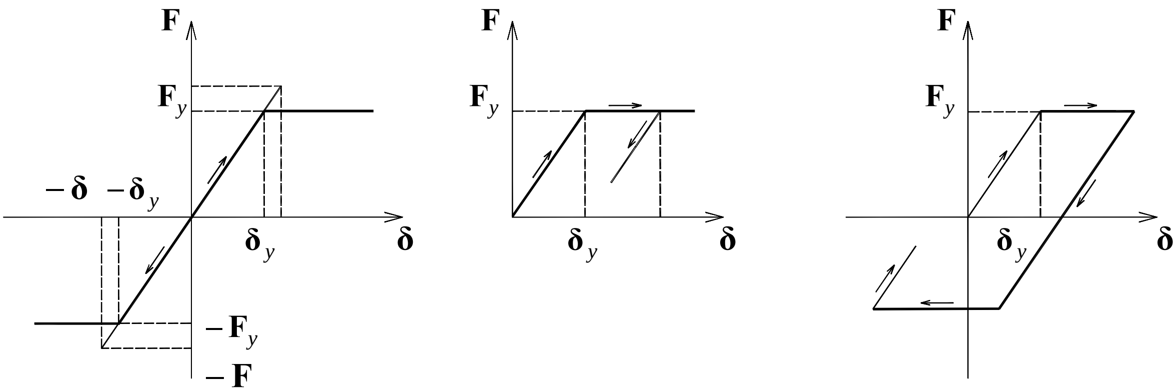


Figure 2 Behaviour states

3. Energy balance for elastoplastic systems

The energy input by earthquakes to elastoplastic systems is dissipated by viscous damping and inelastic deformations.

Integrating the equation of motion of a single-degree-of-freedom system whose mass m is subjected to the external force $-m\ddot{u}_g(t)$, the following equation is obtained:

$$\int_0^u m\ddot{u}(t)du + \int_0^u c\dot{u}(t)du + \int_0^u f_s(u,\dot{u})du = -\int_0^u m\ddot{u}_g(t)du \tag{2}$$

where $f_s(u,\dot{u})$ is the resisting force, depending on the history of the deformation u and on the sign of the velocity $\dot{u}$, and c is the viscous damping coefficient.

The term on the right side of the Eq. (2) is the total energy input to the system, $E_g(t) = -\int_0^u m\ddot{u}_g(t)\,du$. The first term on the left side of Eq. (2) is the kinetic energy of the mass associated with its motion relative to the ground:

$$E_c(t)=\int_0^u m\ddot{u}(t)du=\int_0^{\dot{u}} m\dot{u}(t)d\dot{u}=\frac{m\dot{u}^2}{2} \quad (3)$$

The second term on the left side of Eq. (2) is the energy dissipated by viscous damping, $E_a(t)=\int_0^u c\dot{u}(t)du$. The third term on the left side is the strain energy $E_{ep}(t)=\int_0^u f_s(u,\dot{u})du$, that is the sum of the energy dissipated by inelastic deformations, $E_{pl}(t)$, and the reversible strain energy of the system, $E_{el}(t)$. This last energy being

$$E_{el}(t)=\frac{[f_s(t)]^2}{2k} \quad (4)$$

where k is the initial stiffness of the elastoplastic system, the energy dissipated by inelastic deformations is

$$E_{pl}(t)=E_{ep}(t)-E_{el}(t)=\int_0^u f_s(u,\dot{u})du-\frac{[f_s(t)]^2}{2k}$$

Based on these definitions, Eq. (2) represents the energy balance for the elastoplastic system,

$$E_g(t)=E_c(t)+E_a(t)+E_{el}(t)+E_{pl}(t) \quad (5)$$

The energy quantities from Eq. (5) can be computed expressing the integrals with respect to time. Thus, the damping energy is

$$E_a(t)=\int_0^t c[\dot{u}(t)]^2 dt$$

and the yielding energy is

$$E_{pl}(t)=\left[\int_0^t \dot{u} f_s(u,\dot{u})dt\right]-E_{el}(t)$$

The kinetic energy E_c, and the elastic strain energy E_{el}, can be computed directly from Eqs. (3) and (4), respectively.

For multi-degree-of-freedom systems, the energy quantities from the energy balance equation (5) are computed as the sum of contributions of all the structural members.

In a time-history analysis, the structure is subjected to cyclic loadings. In every cycle of loading, the earthquake input energy expressed as the work done by the inertia forces through the relative displacements of the structure, is dissipated through a quantity proportional with the hysteretic loop.

At the analysis time t_{i+1}, the strain energy is:

$$E_{ep}^{(i)}=\sum_{t_1=0}^{t_{i+1}}\sum_{e=1}^{nel}\left(\Delta E_{ep,e}\right)_i$$

For a finite element,

$$\left(\Delta E_{ep,e}\right)_i=\left(\Delta\boldsymbol{\delta}_e^T\right)_i\mathbf{F}_i+\frac{1}{2}\left(\Delta\boldsymbol{\delta}_e^T\mathbf{k}_e\Delta\boldsymbol{\delta}_e\right)_i$$

is the variation of the strain energy on the time step i, $\Delta t=t_{i+1}-t_i$, where:

$\Delta\boldsymbol{\delta}_e$ is the vector of the incremental deformations of the element (lengthening or shortening and incremental rotation of end joints) on the time step Δt;

$\mathbf{F}_i$ is the vector of the end section forces (axial force and bending moment) at the time t_i;

$\mathbf{k}_{e,i}$ is the stiffness matrix of the finite element e at the time t_i, computed according to the element state;

$\left(\mathbf{k}_e\Delta\boldsymbol{\delta}_e\right)_i=\Delta\mathbf{F}_i$ is the force variation in the time step.

If yielding occurs, only the first term, $\left(\Delta\boldsymbol{\delta}_e^T\right)_i\mathbf{F}_i$, is calculated.

At the same analysis time, the elastic strain energy of the system is

$$E_{el}^{(i)}=\frac{1}{2}\sum_{e=1}^{nel}\left(\boldsymbol{\delta}_e^T\mathbf{k}_e\boldsymbol{\delta}_e\right)_i=\frac{1}{2}\sum_{e=1}^{nel}\left(\boldsymbol{\delta}_e^T\mathbf{F}\right)_i=$$

$$=\frac{1}{2}\sum_{e=1}^{nel}\left(\mathbf{k}_e^{-1}\mathbf{F}_i\right)^T\mathbf{F}_i$$

For a finite element, the maximum elastic strain energy is associated with the force values that produce the plastic hinges.

The elastic strain energy evaluated as the work done by the forces through the elastic deformations, $E_{el}^{(i)}$, includes the energy $E_{el}^{(0)}$ associated with the initial gravity loads, at the time t = 0:

$$E_{el}^{(i)}=E_{el}^{(0)}+E_{el,g}^{(i)}$$

$E_{el,g}^{(i)}$ is the reversible strain energy due to seismic action only.
For framed structures, we can write

$$E_{el}^{(0)} = \sum_{e=1}^{nel} \left(E_{el,e}^{(N)} + E_{el,e}^{(M)} \right)_{t=0}$$

where $E_{el,e}^{(N)}$ and $E_{el,e}^{(M)}$ are the energy contributions of the axial force and bending moment, respectively.
For a finite element, when neglecting the shear force effect,

$$E_{el,e}^{(N)} = \frac{1}{2}\boldsymbol{\delta}_e^T \boldsymbol{N} = \frac{1}{2}\boldsymbol{\delta}_e^T \mathbf{k}_e \boldsymbol{\delta}_e = \frac{1}{2}\frac{N^2 l}{EA}$$

and

$$E_{el,e}^{(M)} = \frac{1}{2}\boldsymbol{\delta}_e^T \boldsymbol{M} = \frac{1}{2}\boldsymbol{\delta}_e^T \begin{bmatrix} k_{ii} & k_{ij} \\ k_{ji} & k_{jj} \end{bmatrix} \boldsymbol{\delta}_e = \frac{1}{2}\frac{k_{jj}M_i^2 - 2k_{ij}M_i M_j + k_{ii}M_j^2}{k_{ii}k_{jj} - k_{ij}^2}$$

The earthquake input energy is the work done by the inertia forces through the absolute displacements of the structure,

$$E_g^{(i)} = \sum_{t_1=0}^{t_{i+1}} \left(\mathbf{1}\cdot\Delta u_{g,i}\right)^T \mathbf{M}\left(\ddot{\mathbf{u}}_i + \mathbf{1}\cdot\ddot{u}_{g,i}\right) + \frac{1}{2}\sum_{t_1=0}^{t_{i+1}} \left(\mathbf{1}\cdot\Delta u_{g,i}\right)^T \mathbf{M}\left(\Delta\ddot{\mathbf{u}}_i + \mathbf{1}\cdot\Delta\ddot{u}_{g,i}\right)$$

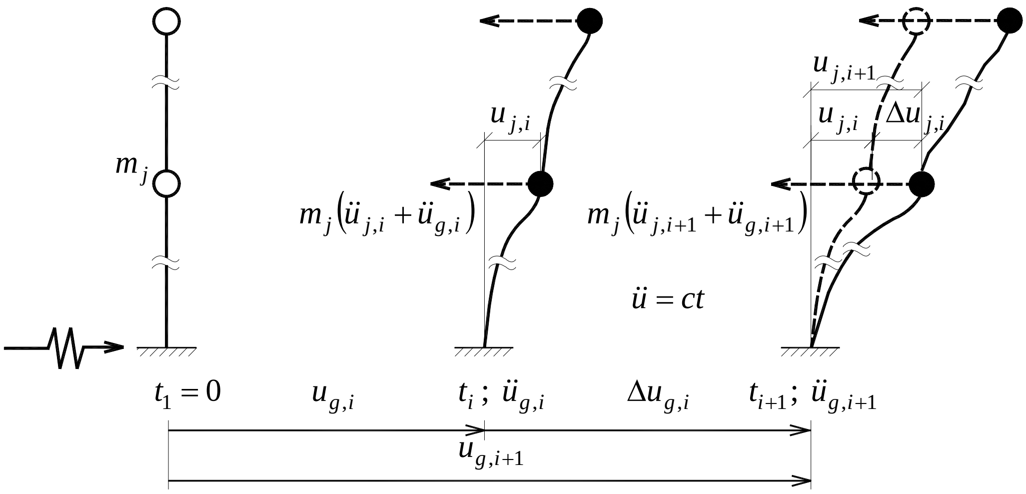


Figure 3 Inertia force variations at the level of the mass m_j

Kinetic energy $E_c^{(i)}$ at the time t_{i+1} is

$$E_c^{(i)} = \frac{1}{2}\left(\dot{\mathbf{u}}_{i+1} + \mathbf{1}\cdot\dot{u}_{g,i+1}\right)^T \mathbf{M}\left(\dot{\mathbf{u}}_{i+1} + \mathbf{1}\cdot\dot{u}_{g,i+1}\right)$$

where the sum of the terms in brackets is the vector of the absolute velocities of the structure at the end of the time step.

The damping energy is expressed as the work done by the damping forces during the time step and is associated with the relative displacements and velocities of the structure:

$$E_a^{(i)} = \sum_{t_1=0}^{t_{i+1}} \left(\Delta\mathbf{u}_i\right)^T \mathbf{C}_i \dot{\mathbf{u}}_i + \frac{1}{2}\sum_{t_1=0}^{t_{i+1}} \left(\Delta\mathbf{u}_i\right)^T \mathbf{C}_i\ \Delta\dot{\mathbf{u}}_i$$

$\mathbf{C}_i$ is the tangent damping matrix, which will be modified at any change of the structure behaviour state. If Rayleigh damping is accepted, then $\mathbf{C} = \alpha_0\mathbf{M} + \alpha_1\mathbf{K}$ and the damping energy can be decomposed in two components, one associated to the constant mass matrix,

$$E_{a,M}^{(i)} = \alpha_0\left(\sum_{t_1=0}^{t_{i+1}} \Delta\mathbf{u}_i^T \mathbf{M}\ \dot{\mathbf{u}}_i + \frac{1}{2}\sum_{t_1=0}^{t_{i+1}} \Delta\mathbf{u}_i^T \mathbf{M}\ \Delta\dot{\mathbf{u}}_i \right)$$

and the other associated to the structural stiffness matrix, that may suffer modifications at the forming or closing of the plastic hinges,

$$E_{a,K}^{(i)} = \alpha_1 \sum_{t_1=0}^{t_{i+1}} \left[\frac{1}{2}\sum_{e=1}^{nel} \left(\Delta E_{a,k,e}\right)_i \right]$$

$\left(\Delta E_{a,k,e}\right)_i$ is the damping energy variation for a finite element *e*, in the time step Δt:

$$\left(\Delta E_{a,k,e}\right)_i = \left(\Delta \boldsymbol{\delta}_e\right)_i^T \left(\mathbf{k}_e \cdot \dot{\boldsymbol{\delta}}_e\right)_i + \\ + \frac{1}{2}\left(\Delta \boldsymbol{\delta}_e\right)_i^T \left(\mathbf{k}_e \cdot \Delta\dot{\boldsymbol{\delta}}_e\right)_i$$

At any time t_{i+1}, the equilibrium condition is expressed by the energy balance equation,

$$E_g^{(i)} - E_c^{(i)} - E_a^{(i)} - E_{ep}^{(i)} + E_{el}^{(0)} = 0$$

The earthquake input energy, and the energy dissipated by yielding and/or damping are indicators of the member and structure ductility.

4. Ductility and behaviour factor *q*

Present seismic building codes consider the structure ability to undergo inelastic deformations, i.e. the structure ductility capacity. Ductility is a quantity not precisely defined, depending on the structure type, structure material, details of the members and their connections, structural redundancy, intensity of the gravity loads, etc. Ductility may be considered at the structure, element or material level. The acceptance of only limited inelastic deformations (in order to avoid the damage of the structure) permits the reduction of the seismic forces associated to an elastic behaviour, by dividing them to the behaviour factor *q*.

The correct evaluation of the behaviour factor ensures the seismic resistance of the structures designed with the capacity design method, if the members and connections are properly detailed and constructed, in order to permit the structure to attain the ductility level and the capacity of dissipating the input energy.

Estimation of the safety factor under severe earthquakes depends on the definition of the collapse conditions or the allowable structural damage level. Besides the probabilistic nature of the damages that a structure in a seismic area may suffer during its lifetime, there is the uncertainty about the material and element strength, and about the ground motion too.

The ductility concept should also consider the number of plastic excursions and, implicitly, the dissipation capacity of the energy input by the earthquake. In this case, a useful calculation instrument is the energetic approach based on energy balance (H. Akiyama: *Earthquake-Resistant Limit State Design for Buildings*, University of Tokyo Press, 1985). From the energy point of view, collapse occurs due to the inability of the structure to dissipate the input energy.

Global ductility is a parameter of quick estimation of the structural damage. The most detailed information about damage is offered by individual members, through their plastic deformations attained during the cyclic loadings, and is expressed by equivalent ductility factors μ. The concept of the equivalent ductility factor accounts for the influence of the cyclic response, unlike the conventional procedure where the ductility demand is evaluated based on pushover analysis.

Nowadays, the evaluation of the behaviour factor *q*, that must express the global behaviour of a building under a severe seismic action, does not have a consolidated base. The correct evaluation, as the ratio between the ground acceleration value leading to collapse and the value of the ground acceleration when the first plastic hinge is formed, requires a great number of nonlinear dynamic analyses to be performed, for different ground motions. In order to eliminate this computational effort, simplified methods of evaluation of the *q*-factor have been developed by many researchers, that can be grouped in three categories [2]: methods based on ductility factor theory; methods based on the extension of the results obtained for SDOF with inelastic dynamic response; methods based on the energy approach.

Studies on the low-cycle fatigue phenomenon have provided intermediate results between the kinematic criterion, where ductility is evaluated as a displacement ratio, and the energy criterion based on the global hysteretic ductility, which produces the severest results (Krawinkler and Nassar, 1992).

The energy approach for the evaluation of the behaviour factor *q* represents an extension of the Housner method (1956), and has the advantage that the regularity criteria and collapse mechanism assumption do not have to be fulfilled.

In the Como and Lanni method (1983, 1984) [2], the time variation of the nonlinear dynamic response of a structure subjected to seismic action is identified with the energy exchanges that occur in the loading-unloading cycles. Each cycle consists of two phases. During the first phase, the kinetic energy is transformed into reversible strain energy. In the second phase, the energy cumulated in the first phase is transformed into elastoplastic work. The kinetic energy transmitted by the ground in motion is neglected in this last phase. Under severe earthquakes, inertia forces increase from 0 to $\eta\ \mathbf{P}$, when the first plastic hinge or the global collapse mechanism occurs. The seismic resistance of a structure depends on its strength capacity, expressed by the η-factor, and its ductility. Based on these reasons, and removing the assumption of simultaneous formation of all plastic hinges, the behaviour factor q represents an amplification factor for the design earthquake accelerogram, or a reduction factor of the severest earthquake accelerogram. The behaviour factor can be calculated as

$$q = \sqrt{\frac{E_{el-pl}}{E_{el,c}}} \qquad (6)$$

where $E_{el,c}$ is the reversible strain energy until the first plastic hinge is formed, and E_{el-pl} is the sum of the reversible strain energy and the yielding energy, until the global mechanism is formed. The energy quantities $E_{el,c}$ and E_{el-pl} can be approximately evaluated through the work done by the equivalent seismic forces statically applied, distributed according to a combination of vibration modes. q-factor may be obtained by a pushover analysis or, more realistically, a nonlinear dynamic analysis.

5. Case-study. 15-story eccentrically braced steel-framed building

Seismic response of a 15-story steel structure of an office building located in Bucharest is investigated. The worked example is referred to the structural Eurocodes 3 [3] and 8 [4], [5], which will be implemented in the Romanian seismic code under elaboration, P100-2004.

The structure is made up of a central core, which resists the horizontal forces produced by the seismic action, and a subsystem formed by perimeter columns, that resists only the corresponding gravity loads. Four eccentrically braced frames, with rigid connections, compose the central core. The joints of the beams connecting the perimetral columns to the central core and to each other are pinned (Fig. 4, *a*). The floors are composite slab type. The inner and outer walls are light. Steel Fe 360 and Fe 510 are used.

For eccentrically braced frames designed with the capacity design method, EUROCODE 8 recommends for the behaviour factor the value $q = 5 \cdot 1.1 = 5.5$, where 1.1 is the overstrength default value [4].

The behaviour of the structure was verified by the time-history analysis method, using the recorded accelerograms at INCERC-Bucharest, of the following Vrancea earthquakes: March 4, 1977, N-S component; August 30, 1986, N-S component; May 30, 1990, E-W component.

Two models of the structure were utilized, one neglecting the structural damping, and the other considering Rayleigh damping

$$\mathbf{C} = \alpha_0 \mathbf{M} + \alpha_1 \mathbf{K}$$

Damping coefficients α_0 and α_1 were calculated based on the critical damping, $\lambda_1 = \lambda_2 = \xi = 2\%$ for the first two vibration modes ($T_1 = 1.848$ sec and $T_2 = 0.605$ sec):

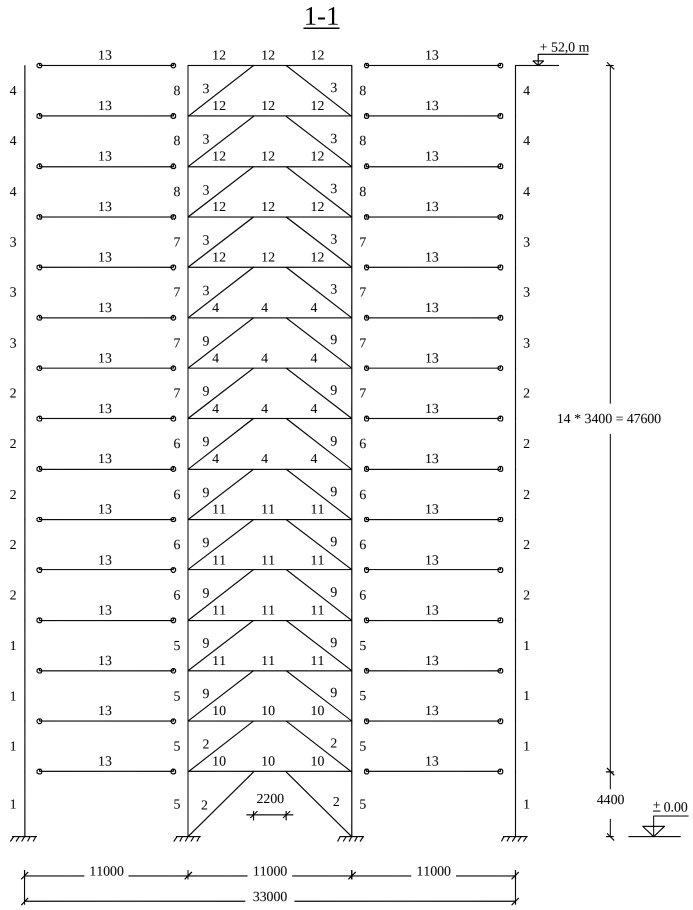
$$\alpha_0 = \frac{4\pi\left(T_2\lambda_2 - T_1\lambda_1\right)}{T_2^2 - T_1^2} = 0.1025$$

and

$$\alpha_1 = \frac{\left(T_2\lambda_1 - T_1\lambda_2\right)T_1T_2}{\pi\left(T_2^2 - T_1^2\right)} = 0.0029$$

Energy quantities associated to the forces and displacements at any time t_i were evaluated for the seismic actions considered in the analysis.

Figures 5 and 6 present the time variation of the energy components for the undamped model and damped model, respectively. The diagrams present also the strain energy associated to the dead loads, $E_{el}^{(0)} = 122.4$ kNm.



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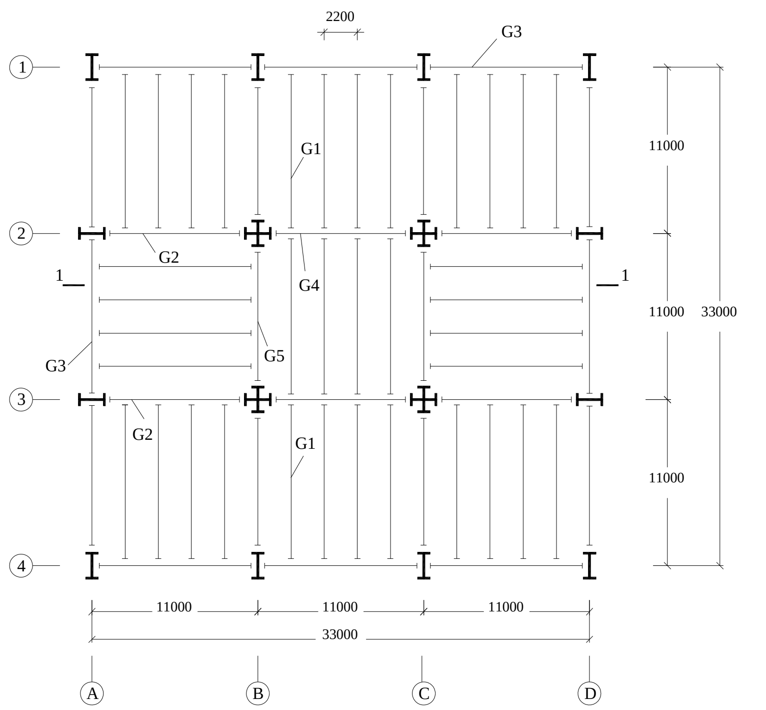
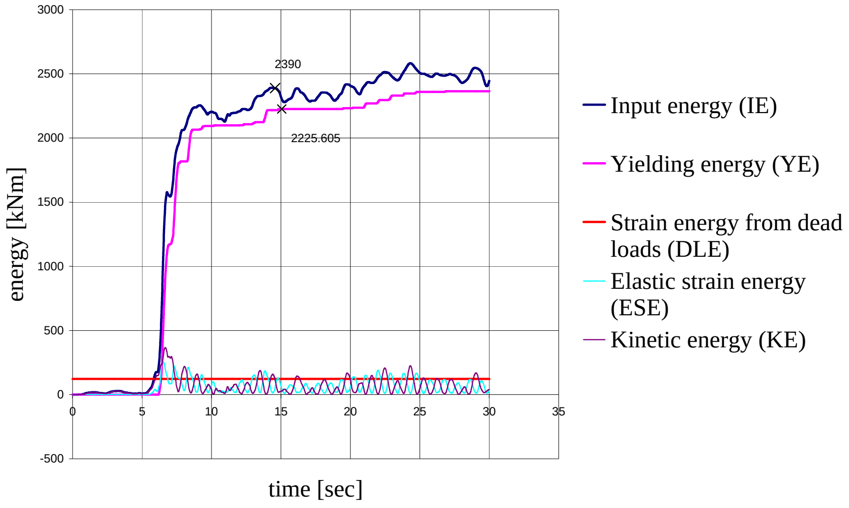


b

Figure 4 *a* – Elevation drawing of the seismic eccentrically braced frame; *b* – Typical floor



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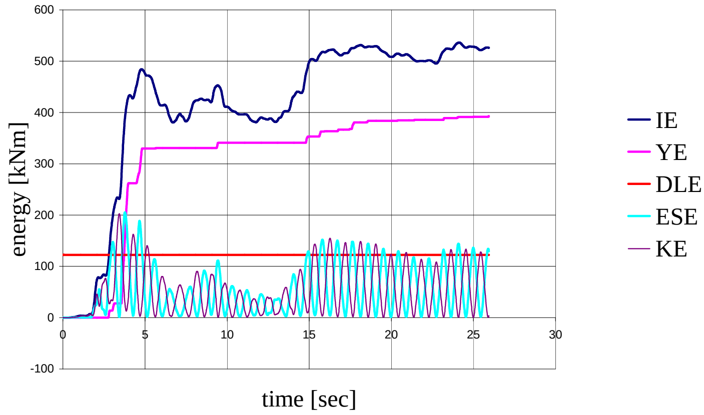

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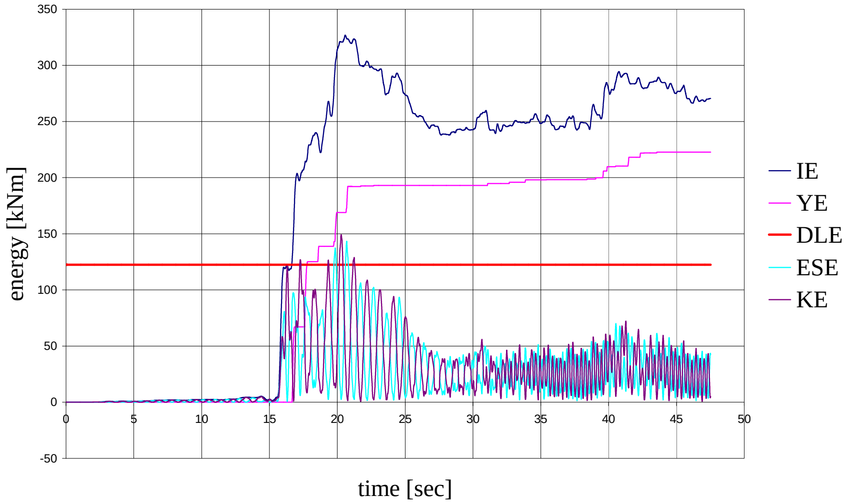
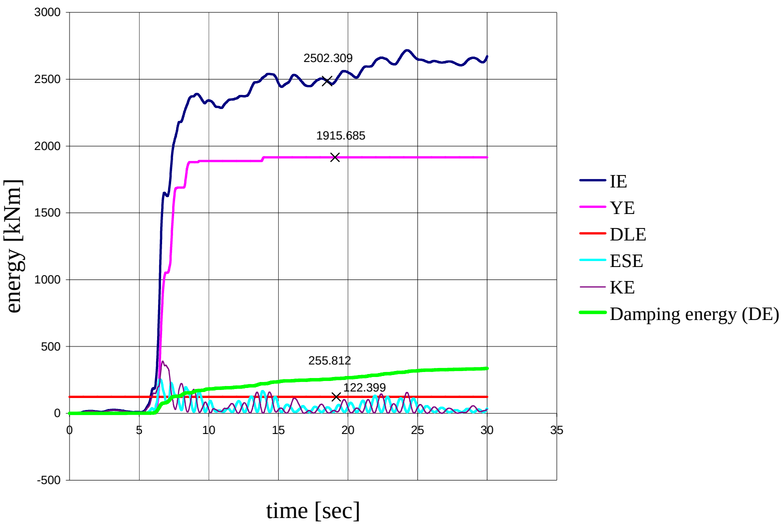


c

Figure 5 Energy time variation. Undamped model.
a - INCERC, 1977, N-S accelerogram. PGA = 1.949 m/sec^2 at t = 6.12 sec
b - INCERC, 1986, N-S accelerogram. PGA = 0.8869 m/sec^2 at t = 3.4 sec
c - INCERC, 1990, E-W accelerogram. PGA = 1.008 m/sec^2 at t = 16.01 sec



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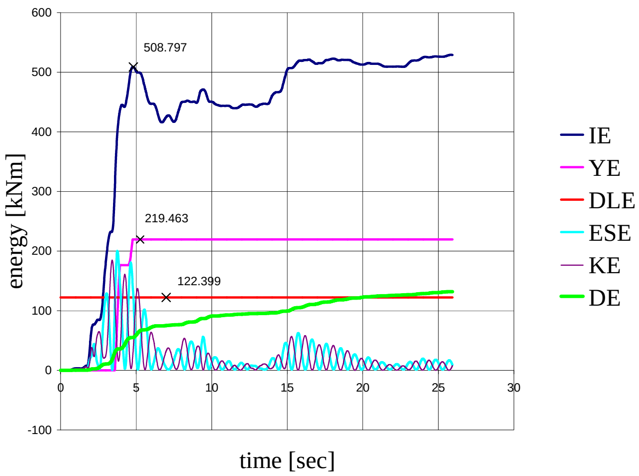

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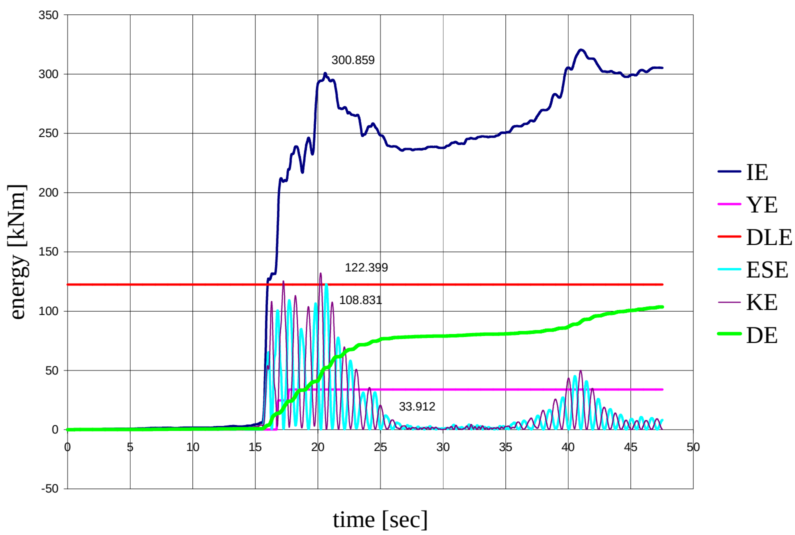


c

Figure 6 Energy time variation. Damped model. *a* - INCERC, 1977, N-S accelerogram; *b* - INCERC, 1986, N-S accelerogram; *c* - INCERC, 1990, E-W accelerogram

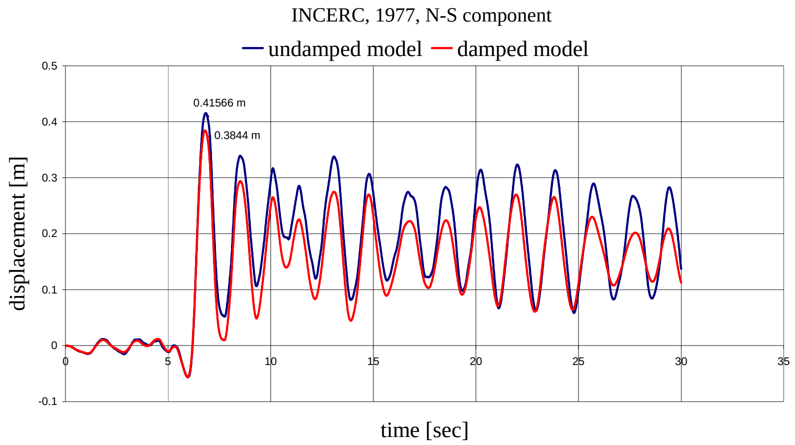


Figure 7 Comparison of lateral displacement at the top of the structure in accordance with the damping consideration

Figures 5 and 6 show that: the earthquake input energy increases as inelastic deformations develop in the dissipative member sections; energy dissipated by inelastic deformations (the plastic hinges rotations) is a very important component in the energy balance; the structural damping reduces the yielding energy, but not significantly; the input energy increases suddenly and is instantaneously dissipated by inelastic deformations (characteristic response at Vrancea earthquakes for the INCERC–Bucharest site).

The effect of the plastic hinges is pointed out by the time variation of the displacement at the top of the structure shown in figure 7. Yielding causes a permanent displacement of 0.2 m when damping is neglected and 0.17 m when damping is considered.

Table 1 presents the maximum (positive and negative) values of the lateral displacements at the top of the structure for the three ground motions, in the absence and the presence of damping, respectively.

A comparison between the components of the balance energy, in accordance with the damping consideration, is presented in table 2, for the INCERC, 1977, N-S accelerogram, at the time t = 15 sec.

Referring to table 2, the energy dissipated by yielding represents 95.9% from the input energy when structural damping is neglected and 77.5% when damping is considered. Therefore, a structure with reduced structural damping subjected to severe ground motions cannot benefit by a significant diminution of the damages produced by the inelastic deformations in dissipative zones.

Figure 6, *a – c* shows a continuous increase of the damping energy, while the plastic excursions diminish.

The input and yielding energy values presented in table 3 show that the 1977 earthquake is a major one, while the 1986 and 1990 may be considered moderate.

Table 1

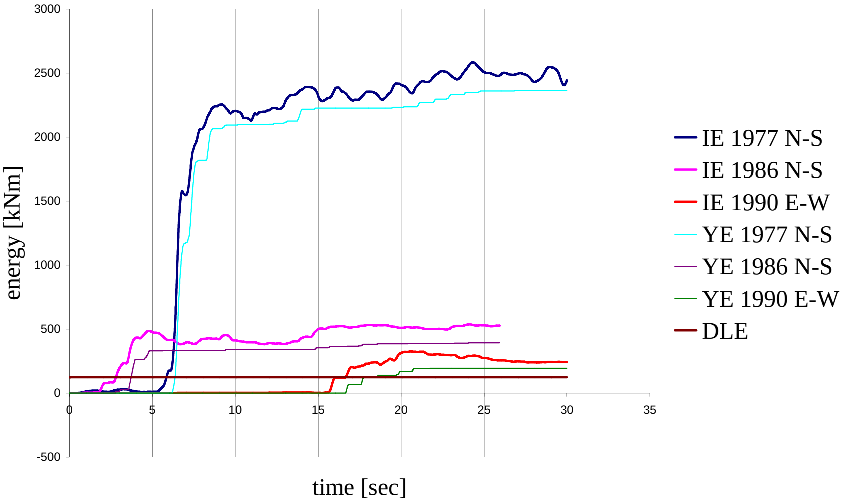
Ground motion	Maximum lateral displacements at the top of the structure [m]			
	undamped model		damped model	
	positive	negative	positive	negative
INCERC, 1977, N-S	0.41566	-0.05639	0.38443	-0.05598
INCERC, 1986, N-S	0.18144	-0.12847	0.17764	-0.11400
INCERC, 1990, E-W	0.10999	-0.11681	0.09364	-0.10980

Table 2

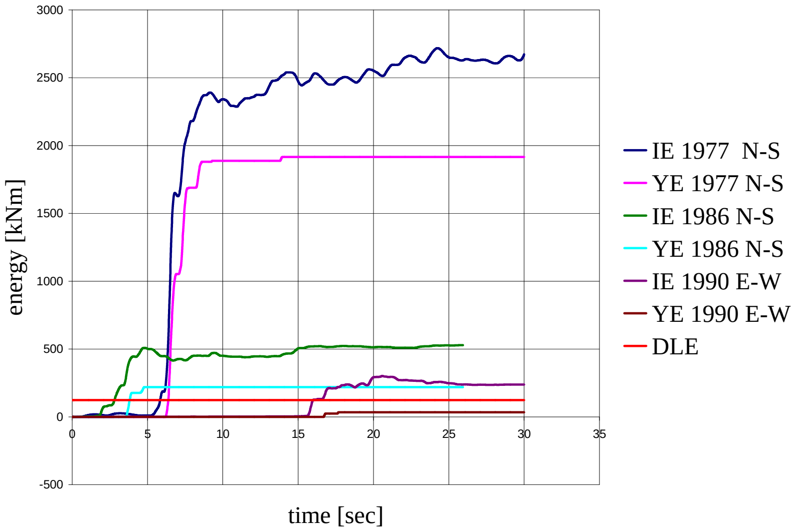
Energy at t = 15 sec [kNm] INCERC, 1977, N-S direction	Structural damping		ε
	No	Yes	
input	2322	2472	+ 6.46%
dissipated by yielding	2226	1916	–13.93%
kinetic	26	28	≅ 0
reversible strain energy	73	56	≅ 0
dissipated by damping ($\mathbf{C} = \alpha_0 \mathbf{M}$)	–	237	

Table 3

Ground motion	Input energy [kNm]		Yielding energy [kNm]		*t* [sec]
	Undamped model	Damped model	Undamped model	Damped model	
INCERC, 1977, N-S	2322	2472	2226	1916	15
INCERC, 1986, N-S	481	505	330	220	5
INCERC, 1990, E-W	325	299	171	34	20.66
INCERC, 1990, N-S	79	–	0.7	–	29.36



a



b

Figure 8 Input and yielding energy time variation for INCERC N-S 1977, N-S 1986 and E-W 1990 ground motions. *a* – Undamped model; *b* – Damped model

The input and yielding energy time variation are presented comparatively for the three ground motions, for the undamped model in Figure 8, *a* and the damped model in Figure 8, *b*. The diagrams contain also the strain energy associated to the dead loads.

6. Conclusions

Evaluation of energy balance components and the diagrams of their time variation permit: to evaluate the severity of a seismic action, according to the value of the input energy; to follow the transfer of the energy into different forms; to assess the seismic structural performance by the distribution of the input energy to the structure elements, whose strength capacity and deformability can thus be controlled.

Estimation of the structural performance requires, not only to locate the sections where plastic hinges develop, but also to control the following quantities:

– plastic rotations and story ductility demand;
– maximum value of the strain energy, E_{el-pl};
– the value of the strain energy stored up to the first yielding, $E_{el,c}$.

Based on these energy quantities, the behaviour factor q_{ef} can be evaluated for the design earthquake, by parametrical studies, for a certain structure type.

For the analyzed structure subjected to INCERC, 1977, N-S accelerogram, when damping is neglected, the behaviour factor is obtained by means of Eq. (6):

$$q_{ef} = \sqrt{\frac{E_{el-pl}}{E_{el,c}}} = \sqrt{\frac{2356}{222}} = 3.26 < q = 5.5$$

The behaviour factor thus calculated represents the ductility demand μ_G imposed by the ground motion, and constitutes a global measure of the structure capacity to dissipate the earthquake input energy.

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