EXPERIMENTAL DETERMINATION OF DYNAMIC CHARACTERISTICS OF STRUCTURES

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Abstract: For building design, the evaluation of dynamic characteristics of the structure, as natural vibration frequency or critical damping coefficient, is a necessary step to obtain the response to certain loads as, for example, the seismic or wind actions. The design characteristics can be different from the real structure characteristics, usually as result of the simplified design models and hypothesis. The experimental determination of the structural dynamic characteristics is important for new buildings and, also, for the existing ones. For new buildings, the experimental values are used to verify the design model and calculations or the build-up of the structures and for an existing structure those tests can be used to establish the present state of the building or to apply certain special retrofitting methods.
This paper presents, by comparison, two experimental methods to estimate the structure dynamic characteristics. The first method is based on structure free vibrations and the second one is based on forced harmonic vibrations induced in the structure. Those methods are exemplified on a small scale model of a structure with two degrees of freedom.
Also, the experimental response of the test structure, subjected to harmonic excitation, is compared with the response of the structure modeled with finite elements.

Keywords: experimental tests, natural frequency, harmonic vibrations, free vibrations, damping ratio

1. Introduction

Computing the dynamic characteristics of the structures is an important step and mandatory, in most cases, in the design of any kind of structure. In the design stage, the calculation of structural dynamic characteristics is based, among others, on the complexity of the resistant structure model and the assumptions concerning the loads and material behavior. All these, combined with differences in materials, manufacturing details and building conditions that may occur in the build-up of the structure, can lead to design dynamical characteristics values more or less similar to their real values.

Performing experimental tests to determine the actual dynamic characteristics of a structure is useful to check the assumptions and the calculation model used in design. However, these experimental tests are not justified for all structures, in economically or technical terms.

In certain circumstances, like structures sensitive to vibrations, in order to apply certain strengthening measures, or when the use of special structural control devices is intended, is appropriate to determine the real dynamic characteristics of structural elements or structure in the ensemble.

Two experimental methods can be applied to determine experimentally, in situ or in laboratory, the dynamic characteristics of a structure, as natural frequency and the damping ratio.. The first method consists in determining the structural response, in terms of displacement or acceleration, at free vibrations produced by a dynamic load, like impact. The second method consists in computing the structural response to harmonic forced vibrations. For information purposes, determining the natural frequencies can also be achieved, by recording and processing the structural response to ambient vibrations with low amplitude.

This paper presents the theoretical background of the first two experimental methods, using the structural response at free vibrations and harmonic forced vibrations. Theoretical aspects are exemplified by experimental tests on a small scale model of a structure with two degrees of freedom, made in laboratory conditions.

2. Theoretical Background

The theoretical background of the dynamic characteristics determination for a certain structure are presented in any paper of dynamics of structures. This paper makes a brief presentation of the information, strictly necessary for computing the dynamic characteristics through experimental methods.

2.1. The Dynamic Characteristics Determination for a Certain Structure Using Free Vibration Method

Free vibration can be induced in a structure by the impact provided by suddenly applying or releasing a given intensity force or by a pulse type movement which excites the structure.

According to the structure or element type, their location and the experimental conditions, free vibrations can be recorded by measuring the structure displacements, the deformation of structural elements or the accelerations in different important points of the structure.

In figure 1 a general response of a structure at free vibrations is represented. The natural period is the time of a full oscillation. Also, the natural period can be computed as the time difference between two consecutive peaks of the structural response, measured in seconds. [1]

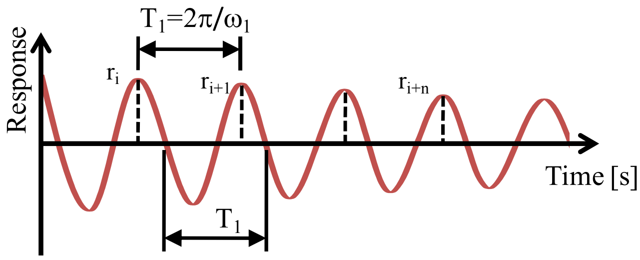

Fig. 1 - Generic response of the structure

The angular frequency ω_1 and the natural frequency f_1 can be calculated with the known natural period determined from the variation graph of the response at free vibrations.

$$\omega_1 = \frac{2\pi}{T_1} \left[\frac{rad}{s}\right] \tag{1}$$

$$f_1 = \frac{1}{T_1} = 2\pi\omega_1 \; [Hz] \tag{2}$$

The first information on the vibration damping level is given by the logarithmic decrement of free vibrations, dependent on the materials used in the build-up of the structure, but also, dependent on the conformation of the structure. The calculation of the logarithmic decrement, δ, is presented in equation (3) as a natural logarithm from the ratio of a two consecutive peaks from the free vibration structure response. If the attenuation of the oscillation is provided slowly, as a result of a reduced damping in the structure, the logarithmic decrement can be computed as an average of several peaks, as presented in equation (4).

$$\delta = \ln\left(\frac{r_i}{r_{i+1}}\right) \tag{3}$$

$$\delta = \frac{1}{n}\ln\left(\frac{r_i}{r_{i+n}}\right) \tag{4}$$

The damping ratio of the system is calculated using the logarithmic decrement of free vibration damping, as follows:

$$\xi = \frac{\delta}{\sqrt{4\pi^2 + \delta^2}} \tag{5}$$

For a reduced damping, usual in case of buildings, the damping ratio can be determined using a rough formula, presented above:

$$\xi \cong \frac{\delta}{2\pi} \tag{6}$$

In figure 2 the difference between the accurate and rough formulas is depicted. This difference is significant if the critical damping ratio is, more or less, over 30%.

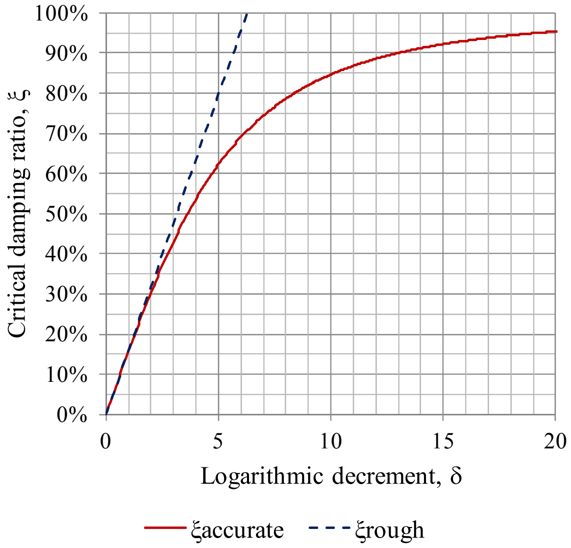

Fig. 2 - Differences between accurate and rough formula for critical damping ratio determination

In free vibrations experimental tests, the logarithmic decrement and the damping ratio are dependent on the amplitude of the structural response. For oscillations with low amplitude, the value of the damping ratio is much smaller than the real value. We have to induce free vibrations with appropriate amplitude for damping ratio determination, which can be made only on an intuitive base. All these provide a high degree of uncertainty to this type of experimental test.

In the study case presented in the paper a comparison between damping ratios is made on the basis of several records of the oscillation, with different amplitudes.

In this regard, the determination of the damping ratio can be made, more accurately, using the harmonic steady state forced vibration method.

2.2. Dynamic Characteristics Determination Using Harmonic Forced Vibration Method.

This method has a higher accuracy than the free vibrations method, but the process is more complicated. Besides measuring devices used to record the structural response, special equipment needed to achieve the excitation of the structural system gives this method a higher complexity. The method required a bigger volume of the processed data than the free vibrations method. With the harmonic forced vibration method we can establish the vibration frequencies of the structure and the damping ratio.

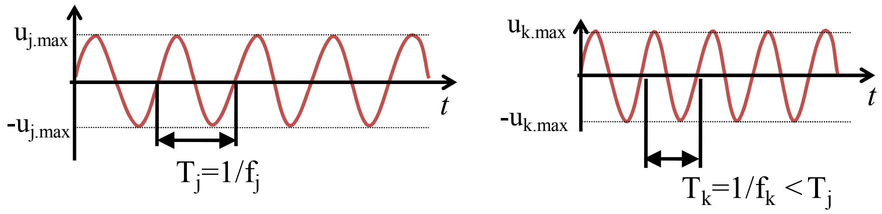

Fig. 3 - Time history displacement for two harmonic excitations

The excitation of the structure is performed using forced harmonic motions, with constant amplitude, which are characterized by different vibration frequencies (figure 3). In order to complete the proposed test bandwidth frequency, one can choose a variation step of the excitation frequency. The maximum response of the structure, corresponding to each excitation movement, is represented on a graph related with the excitation frequency, thus resulting a response curve of the structure to harmonic forced vibrations [2]. One can determine the vibration frequency of the structure corresponding to a given mode of vibration, f_i, and the corresponding vibration period, T_i, by identifying the quasi-resonant frequency, which corresponds to the higher structural response.

$$T_i = \frac{1}{f_i}\ [s] \tag{7}$$

Depending on the width of the frequency band several frequencies can be computed, corresponding to one or more modal shapes. As result of the ongoing technical conditions of the experiment, the excitation oscillation amplitude can vary from one frequency to another. In this way a better representation is the normalized response structure, which is theoretically depicted in figure 4.

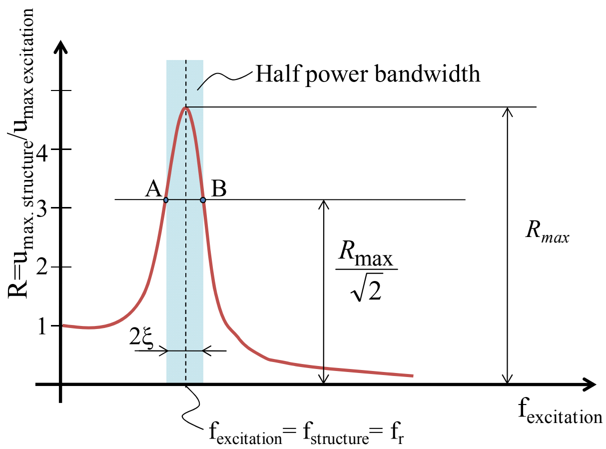

Fig. 4 - Response curve of the structure to harmonic excitation

The damping ratio determination method comes originally from the electrical engineering domain [3] and is called half power bandwidth method. On the response curve at the harmonic forced vibrations one identifies the points A and B, corresponding to the maximum response, multiplied by $1/\sqrt{2}$ (in electrical engineering this value matches to a point where the electric power amplifier is at half of the maximum value). The damping ratio is given by the equation:

$$\xi = \frac{f_B - f_A}{2 f_r} \tag{8}$$

where f_A and f_B are the corresponding frequencies of points A and B, and f_r is the quasi-resonant frequency.

3. Study Case

The case study was made on a small scale model of a structure with two degrees of freedom. In figure 5 the test equipment used for the study case is presented.

The system excitation was induced by harmonic forced vibrations, which were performed using a vibrating table, connected to an electromagnetic oscillator. The signal corresponding to harmonic excitation is performed using a signal generator, and it is amplified and transmitted to an electromagnetic oscillator.

The measurement devices are three unidirectional capacitive accelerometers, located on each DOF direction and on the vibrating mass. The electrical signal is amplified by signal conditioners and is transmitted to the data acquisition system. The electrical signal from each accelerometer is converted into acceleration, using a simple transforming function, provided by the accelerometer producer and implemented in the measurement software. The accelerations are recorded and stored in real time using a computer.

Before processing, the recorded accelerations were corrected, using a baseline correction, and filtered, to remove the interferences produced by ambient vibrations or other sources. The correction and the processing were performed with Seismosignal [4], a strong-motion data processing software.

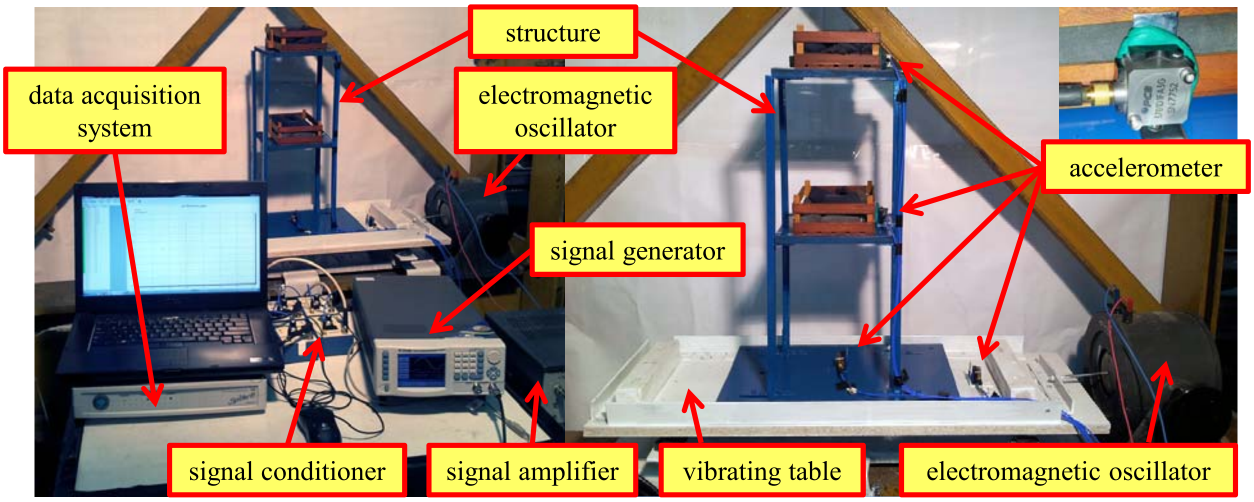

Fig. 5 - Experimental model and testing equipment

3.1 Experimental Results on Free Vibrations

For the free vibration experiment 11 tests were performed. In this tests the accelerations at the top of the structure were recorded, which was manually acted by impact, with different intensity, and after that the structure freely vibrated. In this manner the records of the structure response with different amplitudes were obtained. In figures 6 and 7 two records of the structure's free vibrations are depicted, with different amplitudes of accelerations, and, also, the graphical processing of the results.

In order to obtain the same algorithm to determine the dynamic characteristics of the structure for all records, we considered a constant number of values to be processed. Thus, to determine the vibration period and logarithmic decrement the first seven acceleration peaks were considered. The proper period of the structure was calculated as an average of the first six time intervals between the peak values of acceleration, considering either positive or negative acceleration, depending on which ones were greater.

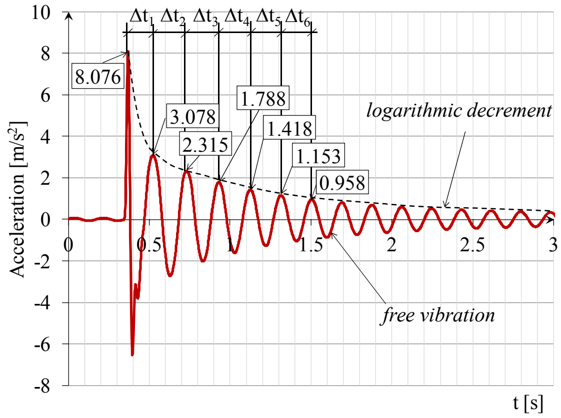

Fig. 6 - Free vibration of the structure in test 8

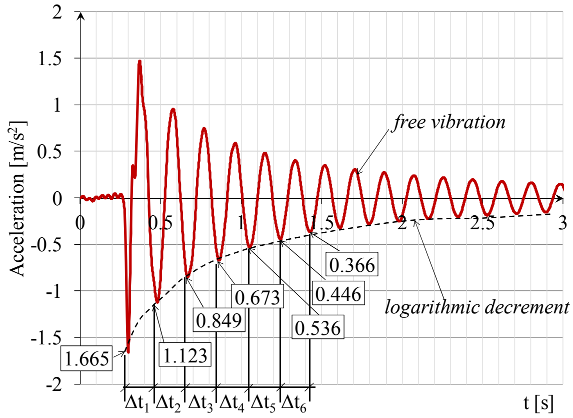

Fig. 7 - Free vibration of the structure in test 1

$$T_1^{test\ i} = \frac{1}{6}\sum_{k=1}^{6}\Delta t_k^{test\ i} \qquad [s] \tag{9}$$

$$f_1^{test\ i} = \frac{1}{T_1^{test\ i}} \qquad [Hz] \tag{10}$$

$$\delta^{test\ i} = \frac{1}{6}\ln\left(\frac{\ddot{u}_1}{\ddot{u}_7}\right) \qquad [Hz] \tag{11}$$

The dynamic characteristics of the two DOFs system are centralized in table 1.

Table 1

Dynamical characteristics of the structure from free vibration tests

Experimental test	Test 1	Test 2	Test 3	Test 4	Test 5	Test 6	Test 7	Test 8	Test 9	Test 10	Test 11
T_1 [s]	0.191	0.188	0.188	0.198	0.198	0.202	0.202	0.189	0.201	0.207	0.196
f_1 [Hz]	5.245	5.310	5.319	5.051	5.046	4.963	4.959	5.286	4.983	4.831	5.098
δ	0.272	0.252	0.254	0.377	0.383	0.402	0.407	0.355	0.433	0.287	0.251
ξ_{rough} [%]	4.33%	4.02%	4.04%	6.00%	6.10%	6.41%	6.48%	5.65%	6.89%	4.57%	4.00%
$\xi_{accurate}$ [%]	4.32%	4.02%	4.04%	5.99%	6.09%	6.39%	6.46%	5.65%	6.88%	4.56%	4.00%

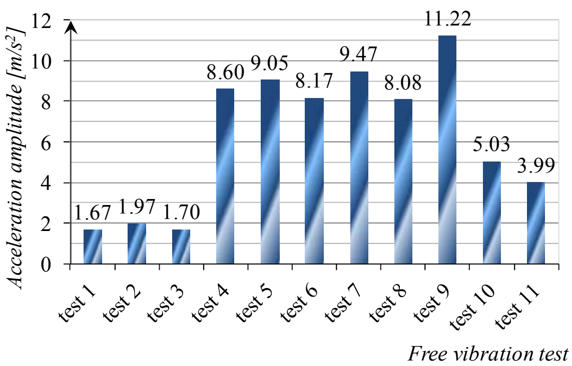

Fig. 8 - Maximum acceleration for the free vibration tests

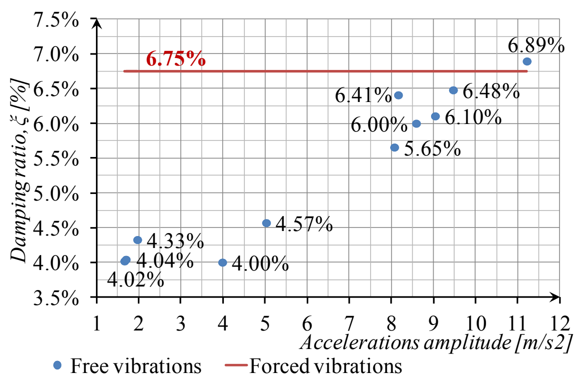

Fig. 9 - Comparison of damping ratio values related to the response amplitude of structure at free vibrations

To exemplify the observation made in paragraph 2.1, regarding the dependency between the computed damping ratio and the amplitude of free vibration oscillation, the peak acceleration in the experimental tests had different values, which are presented in figure 8. After processing the records, a large spectrum of the damping ratio values were obtained. In figure 9 the relation between the computed damping ratios and the peak acceleration of the free vibrations is depicted and compared with the computed damping ratio obtained from forced vibrations, which are presented in the next paragraph. One can observe that for small amplitudes of the oscillation the resulted damping ratio is much smaller than the value of the damping ratios obtained for large acceleration amplitudes and particularly as compared with the damping ratio from the forced vibrations, considered in the technical literature as being much more accurate.

3.2 Experimental Test Results for the Steady State Forced Harmonic Excitation

For the experimental test at forced vibrations, the structure was acted, through the vibrating table, with a series of harmonic excitations with different vibration frequencies. The considered frequencies bandwidth was from 1 Hz to 10 Hz, with a frequency step of 0.1 Hz, between 2 Hz and 6 Hz.

To exemplify the excitation and, also, the response of the structure, in figure 10 the recorded accelerations at the top of the structure and at the level of the vibrating table are presented. In this case, the frequency of the oscillation is 5.1 Hz.

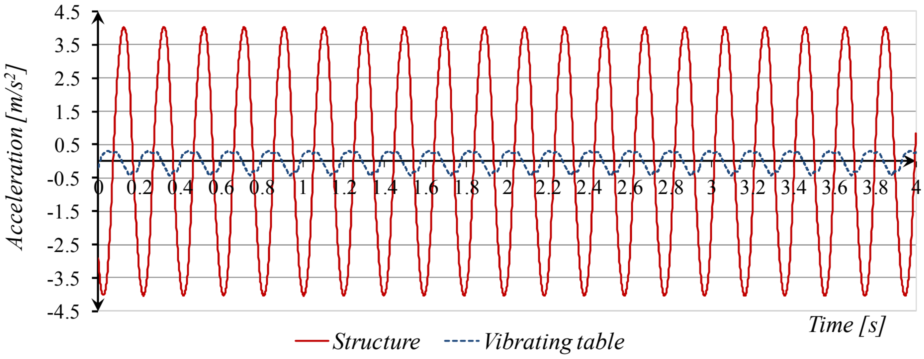

Fig. 10 - Recorded acceleration for 5.1 Hz harmonic excitation

For every frequency from the considered bandwidth, were determined the peak accelerations at the top level of the structure and at the vibrating table level. The structural response was considered as a ratio between the previous described values. Figure 11 depicts the normalized response of the structure related to the excitation frequency. One can observe that the maximum amplification, matching with a quasi-resonance phenomenon, was obtained for a 5.1 Hz frequency. The resonant frequency represents the natural frequency of the structure.

The natural period is computed from the inverse ratio:

$$T_1 = \frac{1}{f_1} = \frac{1}{5.1} = 0.196\,\text{s} \tag{12}$$

For the calculation of the damping ratio the half power bandwidth method is applied. To simplify the calculus, the response curve of the structure is depicted, this time, in relation with the normalized frequency, obtained as a ratio between the frequency of the excitation and the resonant frequency. In this graph, the half power response is computed by multiplying the maximum response with $1/\sqrt{2}$. In this manner, the intersection points of the response curve and the horizontal line, corresponding to $9.56/\sqrt{2} = 6.76$, are obtained. The normalized frequencies corresponding to the intersection points are 0.925 and 1.06, visually identified on the graph from figure 12.

The damping ratio result, as following:

$$\xi = \frac{1.06 - 0.925}{2} = \frac{0.135}{2} = 0.0675 \quad (6.75\%) \tag{13}$$

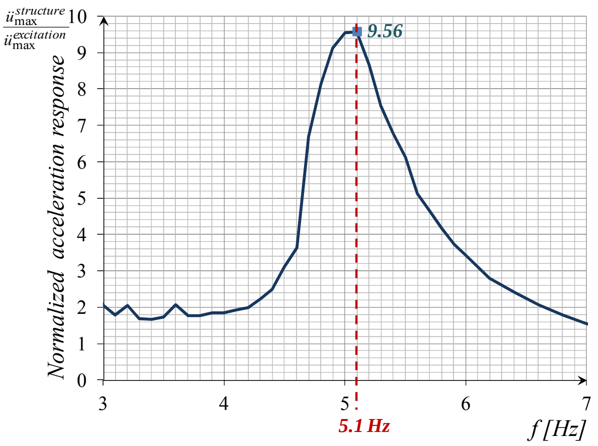

Fig. 11 - Normalized structural response vs. excitation frequency

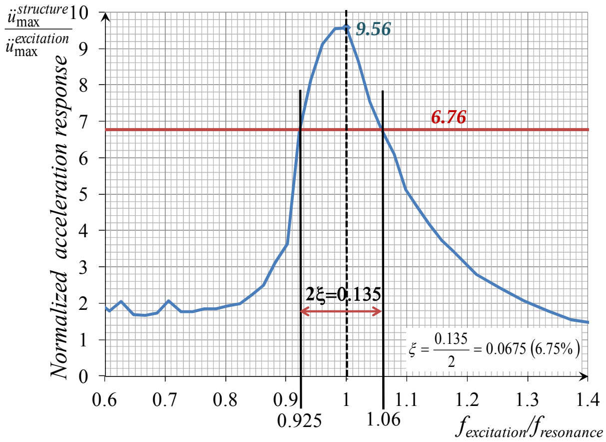

Fig. 12 - Half power bandwidth method

3.3. Comparison of the Experimental Structural Response with FEM

To appraise the accuracy of the experimental test results a numerical analysis regarding the response of the structure to forced harmonic vibrations was computed. The two DOF structure was modeled with finite elements in SAP 2000 [5]. The FEM model was made to comply with the real laboratory model. The mass disposition on every level of the structure, the materials types and the connections between elements were implemented in the FEM model as far as possible identical with the real ones. The FEM model and the first shape mode can be observed in figure 13. From the numerical analysis, for the first mode a natural period of 0.19134 s was obtained, corresponding to a vibration frequency of 5.22 Hz, close to the experimental values depicted in table 1.

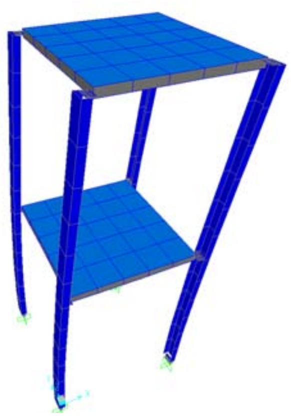

Fig. 13 - Mode 1 of the FEM model

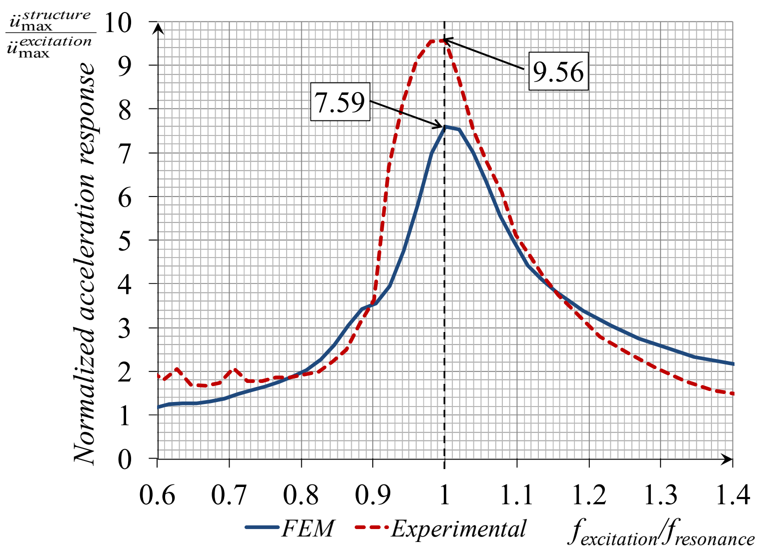

Fig. 14 - Response curves for experimental and FEM models

For the numerical analysis, the input excitation was provided by the recoded acceleration at the level of the vibrating table, filtered and corrected. For the time history analysis a constant modal damping equal with the one determined from experimental tests was considered ($\xi = 6.75\%$).

As result of a small difference between the natural frequencies of the experimental model and the FEM model, for the comparison of the structural response the normalized response curves related to the normalized frequencies was used. In figure 14, the FEM response curve is compared with the experimental response curve.

Considering the response of the model in quasi-resonance state an error can be computed, as follows:

$$err\% = \frac{9.56 - 7.59}{9.56} 100 = \frac{1.97}{9.56} 100 \approx 20\% \tag{14}$$

4. Conclusions

Experimental studies made in laboratory conditions confirmed the theoretical background. As one can see from the experimental test presented in this paper the determination of the dynamic~~al~~ characteristics of a structure using the free vibration method, although the data processing is more simple, the precision is far to be accurate. There are some unknowns, as the oscillation amplitude, the number of processed steps and others, which can induce in the experimental analysis a great indetermination. In some cases, as for example in the dynamic testing of bridges, the free vibration analysis can be considered reliable because of the standardized dynamic load application, provided by experimental testing codes, which conducts to similar testing conditions for similar structures. Also, those load conditions provide a load system very similar to live load on bridges.

A more rigorous method to establish the dynamic characteristics of a structure is the steady state forced harmonic excitation. In this case, a difficult problem is the manner in which the structure is excited to harmonic vibrations. For a real structure, with a greatly value of the mass, the dimension or characteristics of the excitation device are definitively different from the devices used in laboratory conditions. The transportation and placing on the analyzed structure can be difficult and, however, the price of those devices is to be considered.

Related to the verification of the experimental test with FEM, the unique control element between the experimental model and the numerical model was the natural frequency of the system. Taking into account the inherent differences between the two models, the calculated error, around 20%, can be considered an acceptable one.

Taking into account that sometimes the experimental conditions can be limitative, the comparison can be considered, also in the other direction, as a verification of the FEM model with a real behavior of the structure. After such ~~a~~ verification, a numerical extrapolation, with MEF, to other study cases can be more representative.

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