

EVALUAREA PROBABILISTICĂ A COMPORTĂRII UNOR ELEMENTE SIMPLE DE BETON ARMAT

Coordonator științific

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Absolvent

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INTRODUCERE

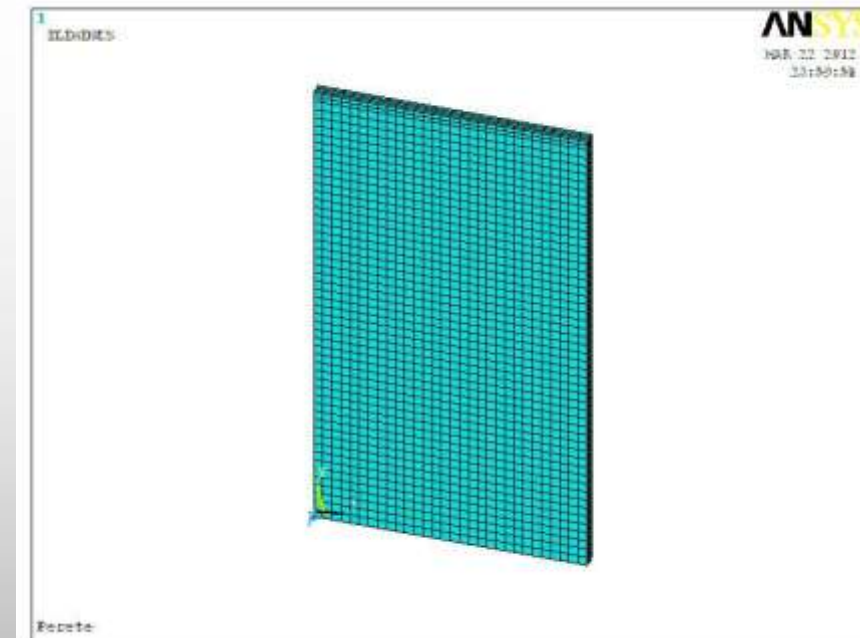
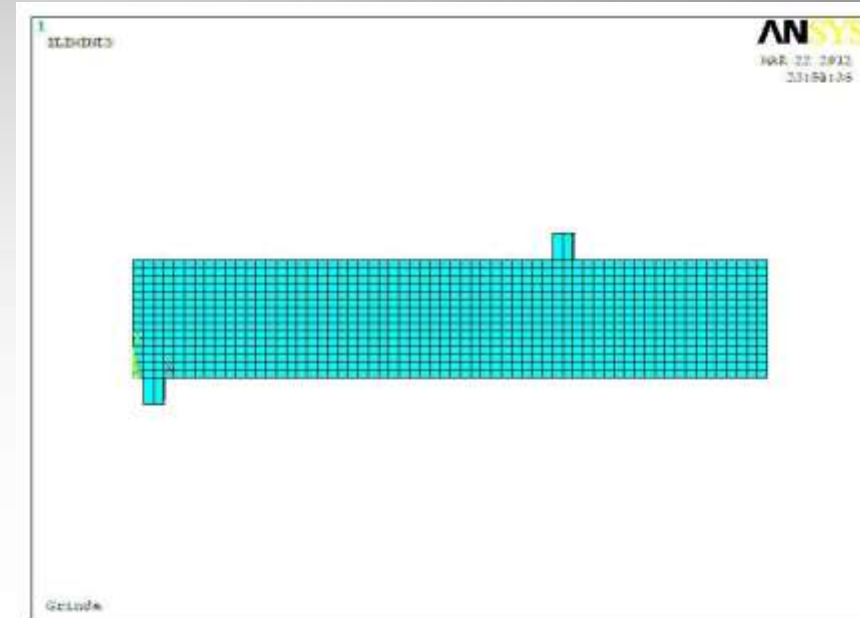
Obiectiv: Evaluarea probabilistica a comportarii unor elemente simple de beton armat .

Studiul a 2 elemente simple de beton armat:

- **Grinda simplu rezemata**
- **Difragma verticala**

Analiza numerica se realizeaza cu ajutorul programului **ANSYS [1]**.

- Cele 2 elemente structurale sunt incarcate cu forte monotone crescatoare si analizate pana la cedare.
- Parametri urmariti in analiza: forta de fisurare, forta la care se manifesta curgerea armaturii, forta ultima si deplasarea maxima



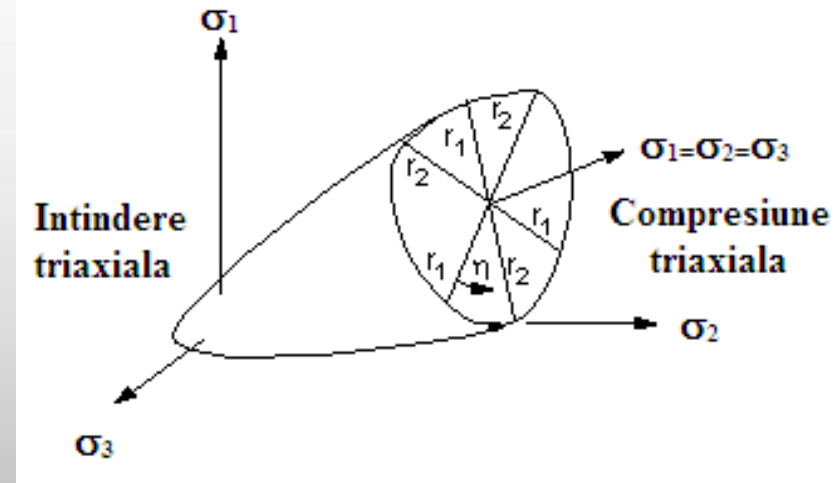
MATERIALE

Materiale componente:

- **Beton simplu**-material fragil
- **Otel**-material ductil

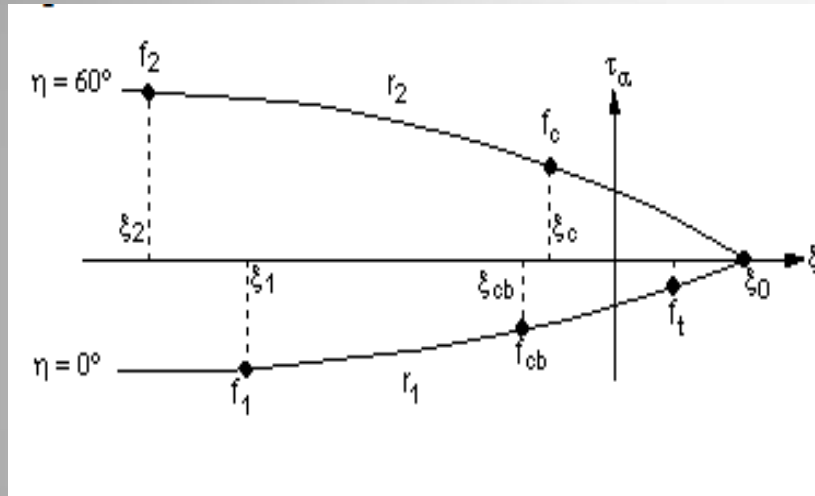
Beton simplu:

- **Criteriul de cedare Willam-Warnke [3]**
- Suprafata de cedare: con cu meridiane curbe si baza necirculara, iar in zona de intindere, forma tetraedrica
- Modelul depinde de 5 parametri: - f_t -rezistența ultimă la întindere monoaxială
 - f_c - rezistența ultimă la compresiune monoaxială
 - f_{cb} -rezistența ultimă la compresiune biaxială
 - f_1 -rezistența ultimă la compresiune biaxială suprapusă pe o tensiune hidrostatică σ_a ;
 - f_2 - rezistența ultimă la compresiune monoaxială suprapusă pe o tensiune hidrostatică σ_a .
- Conditia ce cedare: $f(\sigma_a, \tau_a, \theta)=0$
- Avantaje:-rezultate apropiate de cele experimentale
 - determinare simpla a parametrilor modelului
 - suprafata este continua
 - curbele sunt netede si convexe

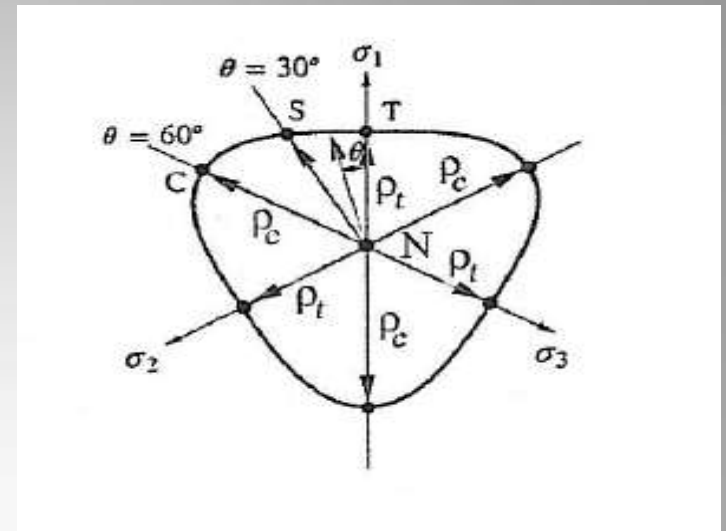


MATERIALE(continuare)

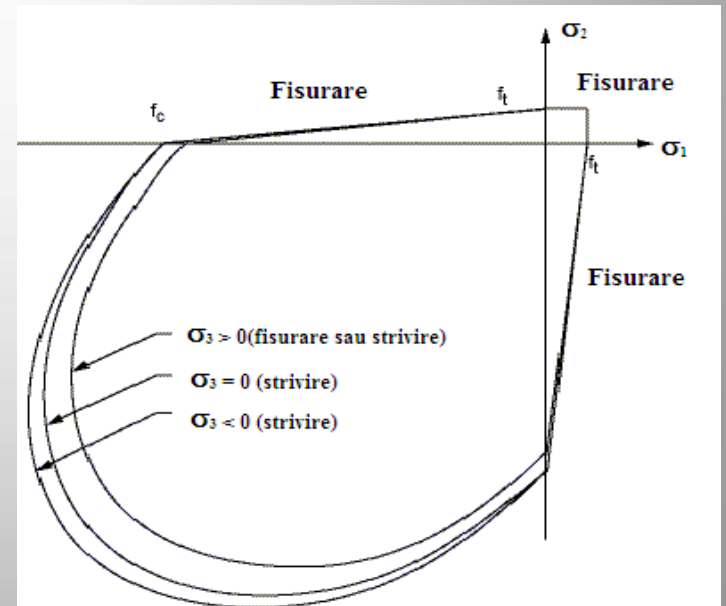
- Din intersecția suprafeței de cedare cu **planul deviator** ($\xi = \xi_0 = \text{constant}$), rezultă o curbă de cedare.



- În cazul în care σ_3 este aproximativ egal cu 0, se poate proiecta în planul σ_1 - σ_2 suprafața de cedare, observându-se posibilitățile de cedare prin fisurare excesivă sau prin strivire



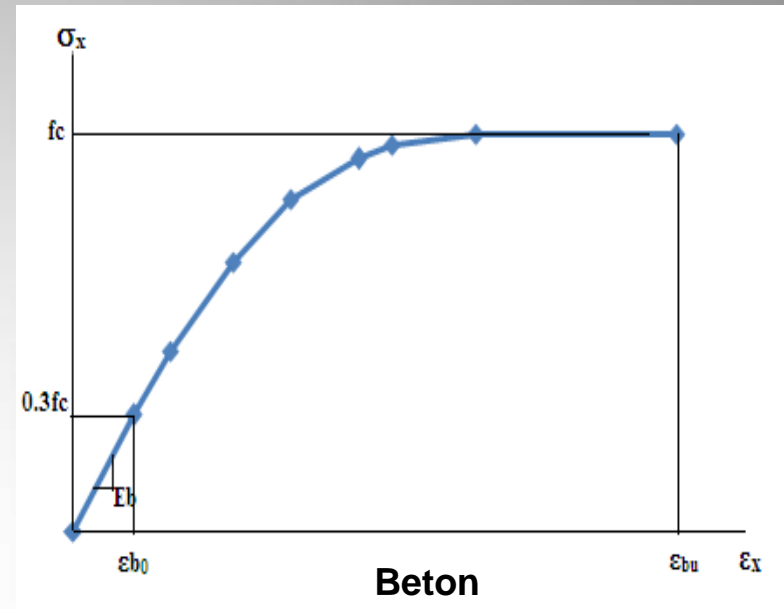
- Curbele obținute din intersecția suprafeței de cedare cu planul $\theta = \text{constant}$ trecând prin axa hidrostatică, se numesc **meridiane**.



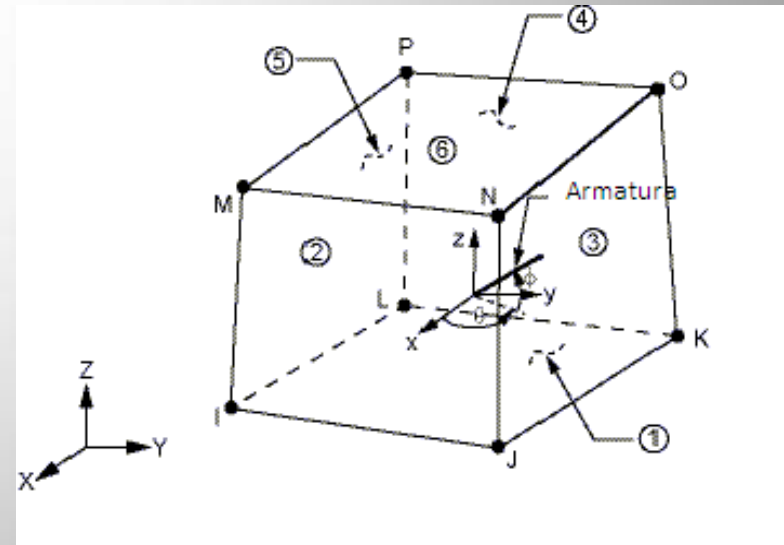
MATERIALE(continuare)

- **Comportarea neliniara a betonului** se modeleaza cu relatiile lui Macgregor:

$$f = \frac{E_c \cdot \varepsilon}{1 + \left(\frac{\varepsilon}{\varepsilon_0}\right)^2}$$
$$\varepsilon_0 = \frac{2 \cdot f_c}{E_c}$$
$$E_c = \frac{f}{\varepsilon}$$



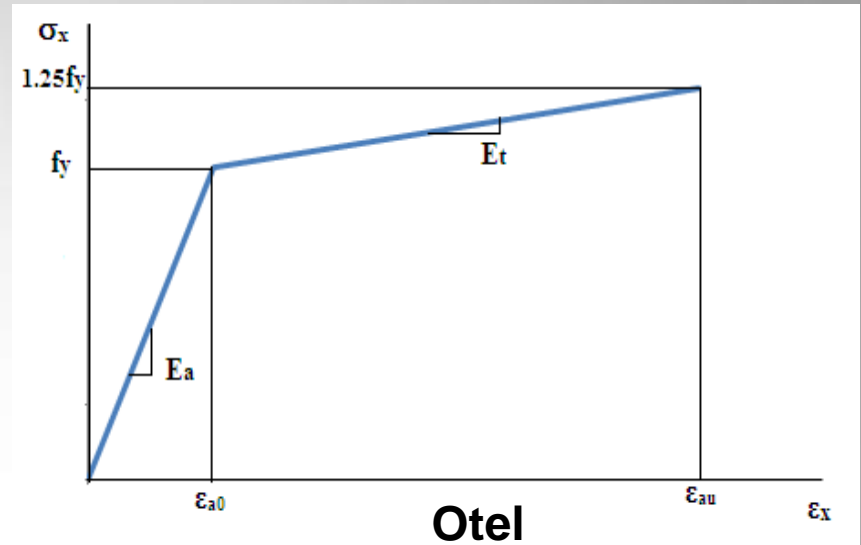
- Pentru discretizarea betonului a fost folosit elementul finit **SOLID 65**.
- SOLID 65 este caracterizat de 8 noduri, cu 3 grade de libertate pe nod.
- Acest element finit este capabil de fisurare in 3 directii ortogonale si rupere prin strivire. [1]



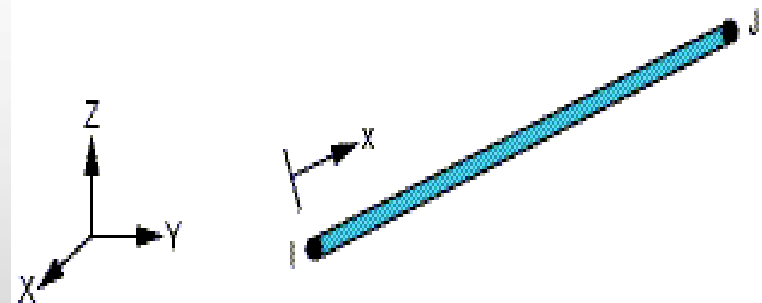
MATERIALE(continuare)

Otel beton:

- Model de comportare biliniar
- Prezinta comportare similara in intindere si compresiune
- Caracterizat de E_a , E_t si f_y



- Pentru discretizarea otelului beton a fost folosit elementul finit **LINK 8**.
- LINK8 este un element de bara 3D axial deformata, cu 2 noduri si 3 grade de libertate pe nod. [1]



GRINDA DE BETON ARMAT

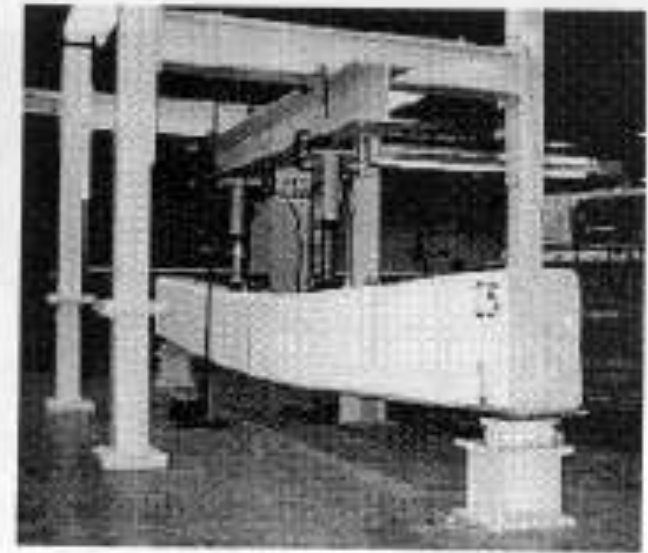
- Este folosita pentru calibrarea modelului, asadar corespunde unui experiment de laborator raportat de Buckhouse [2] in 1997

GRINDA EXPERIMENTALA

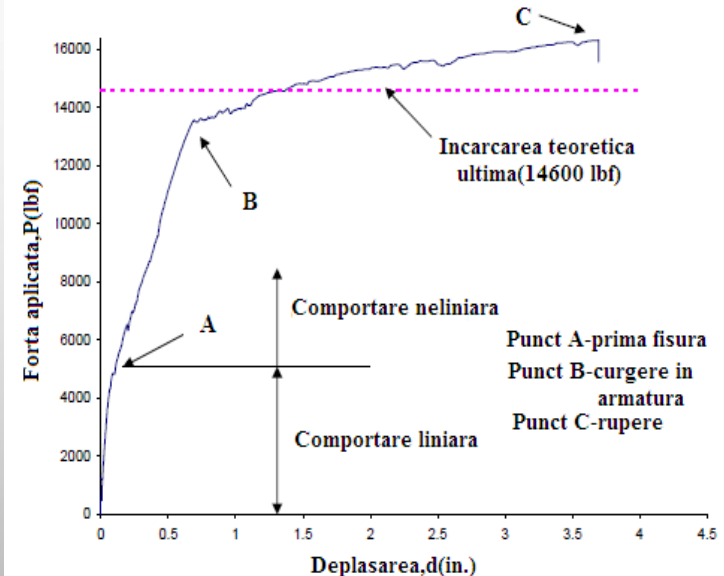
- Dimensiuni: 254x457.2mm, lungime 5638.4mm.
- Armare: 3 Φ 5 armatura longitudinala si etrieri Φ 3/200mm.
- Acoperirea cu beton a armaturii : 50.8 mm.
- Limita de curgere a otelului : $f_y=413.7$ MPa.
- Rezistenta la compresiune a betonului: $f_{cd}=33.1$ MPa.
- Incarcare: 2 forte concentrate monoton crescatoare situate la distanta 1524 mm de reazeme.

Rezultate:

- Primele fisuri : $P=20$ kN
- Intrarea in curgere a armaturii longitudinale : $P= 60$ kN
- Cedarea : $P=73$ kN



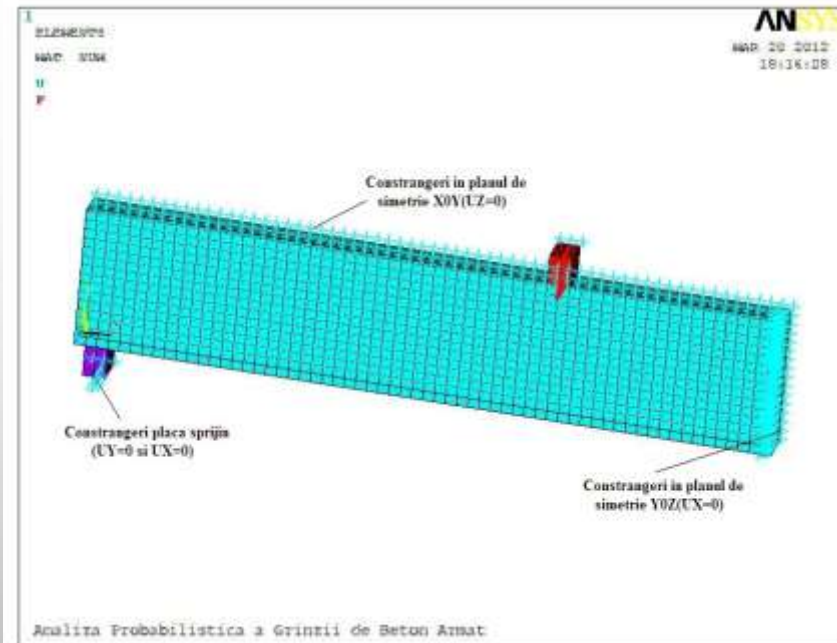
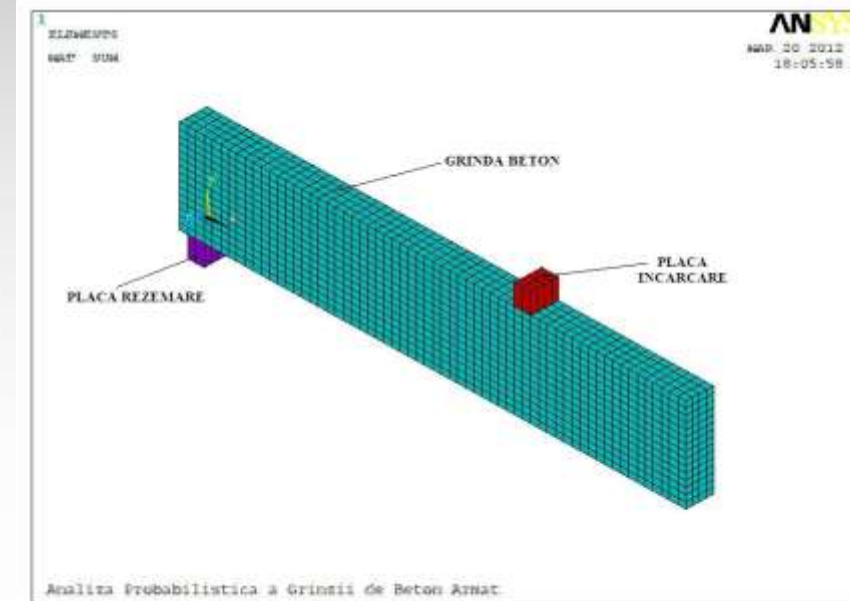
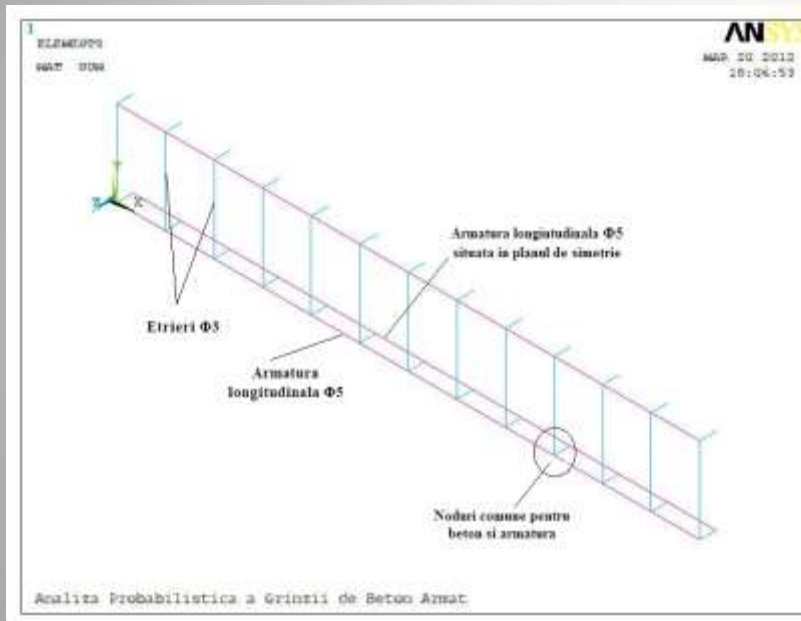
CURBA FORTA-DEPLASARE EXPERIMENTALA



GRINDA DE BETON ARMAT(continuare)

GRINDA ANALITICA

- Datorita simetriei longitudinale si transversale, se modeleaza $\frac{1}{4}$ din grinda
- Elemente finite: beton-SOLID 65,armatura-LINK 8, placi de incarcare si rezemare SOLID 45 [1]

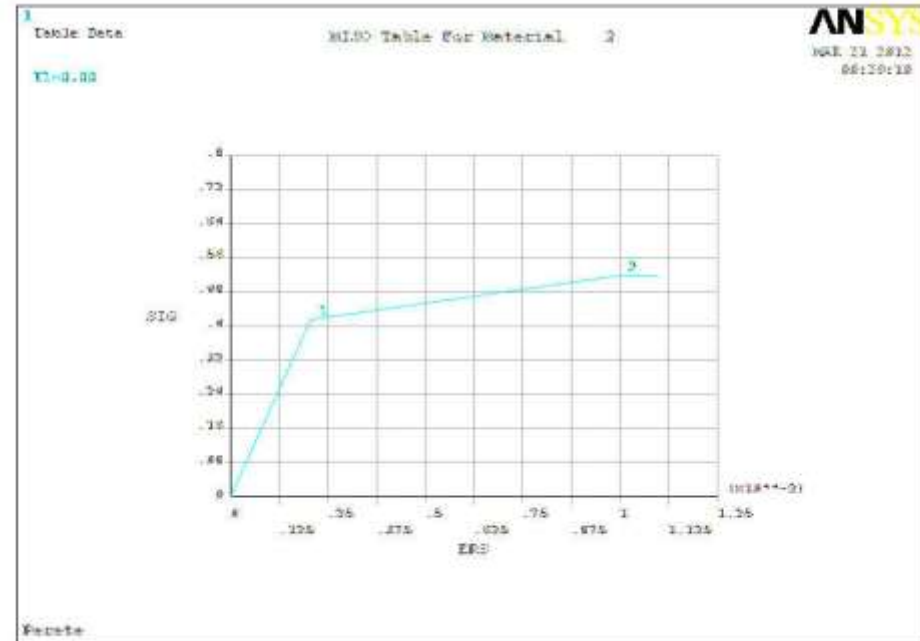


- Pentru a modela simetria sunt introduse conditii de margine in deplasari: $UX=0$ si $UZ=0$
- Placa de sprijin: $UY=0$ si $UZ=0$
- Incarcarea este aplicata concentrat la nodurile placii de incarcare

GRINDA DE BETON ARMAT(continuare)

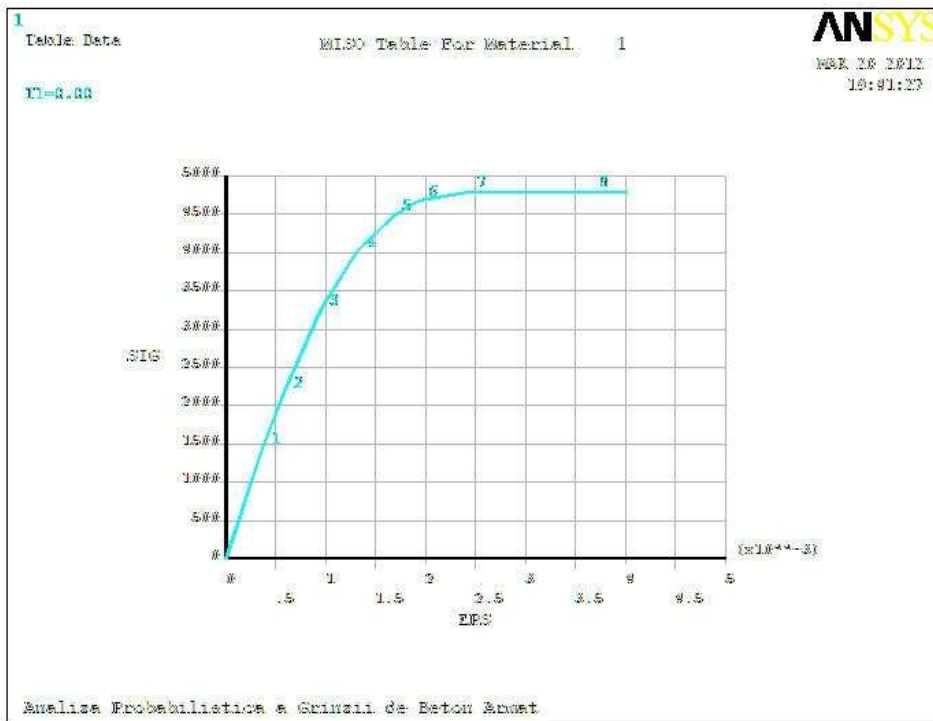
Otel beton

Modul de elasticitate E_a	199900 MPa
Modul tangent E_t	12930 MPa
Efort curgere f_y	413.7 MPa
Deformatia la curgere ϵ_0	0.00207
Deformatia ultima ϵ_u	0.1



Beton

Modul de elasticitate E_b	27230 MPa
Rezistentă compresiune f_c	33.1 MPa
Limita elastică	9.71 MPa
Deformatia elastică ϵ_{b0}	0.00036
Deformatia ultima ϵ_{bu}	0.0036



GRINDA DE BETON ARMAT(continuare)

REZULTATE

Se vor evidenta 3 aspecte:

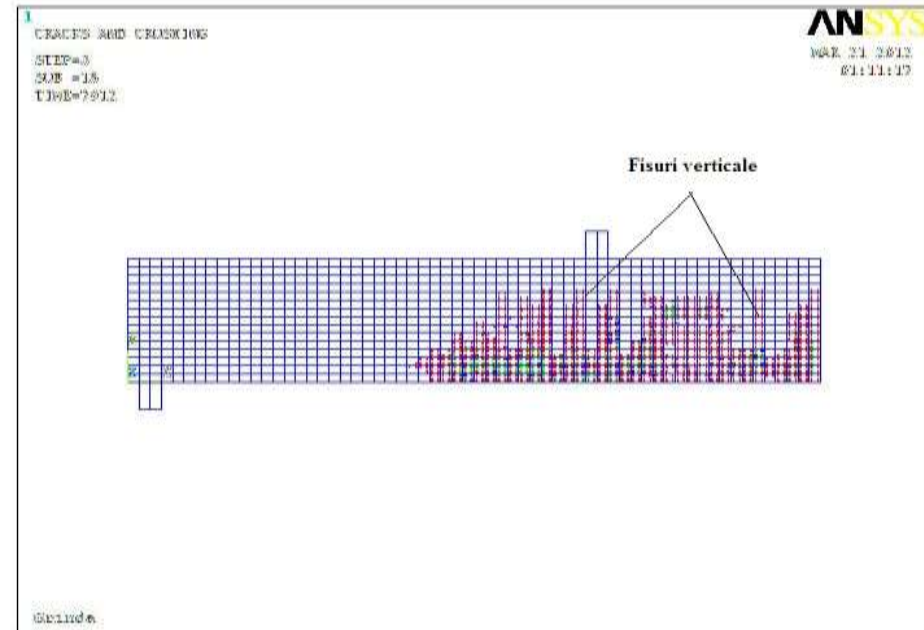
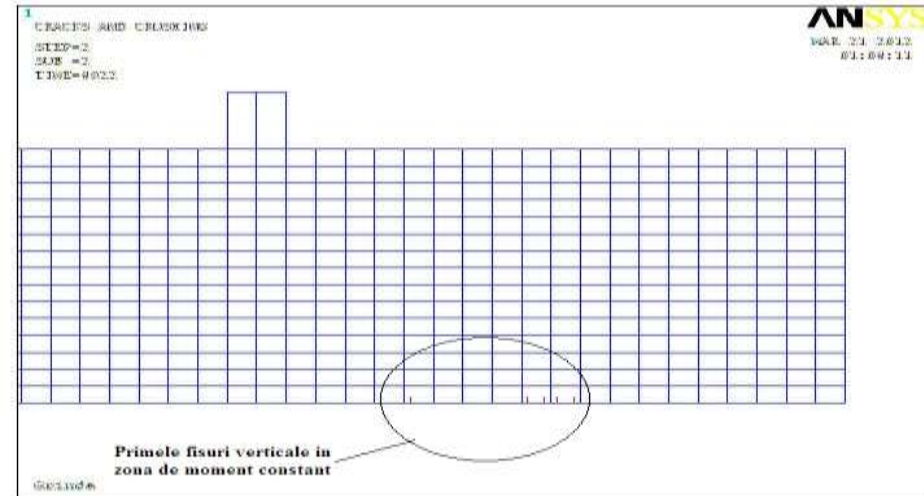
- Fisurarea initiala a grinzii.
- Initierea curgerii in armatura.
- Limita de rupere .

Fisurarea initiala a grinzii

- Fisuri verticale in zona de moment constant la o valoare a incarcarii $P=21.9$ kN
- Testele experimentale au furnizat o valoare $P=20$ kN
- Forta de fisurare calculata manual: $P=23$ kN

Stadiu intermediar fisuri

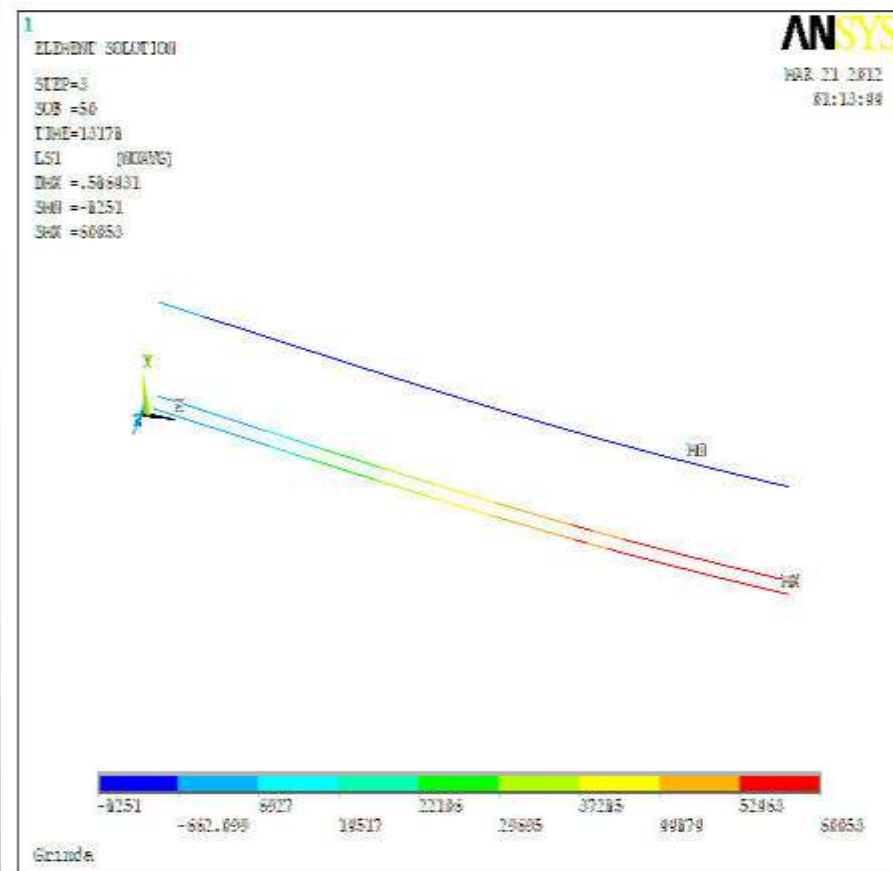
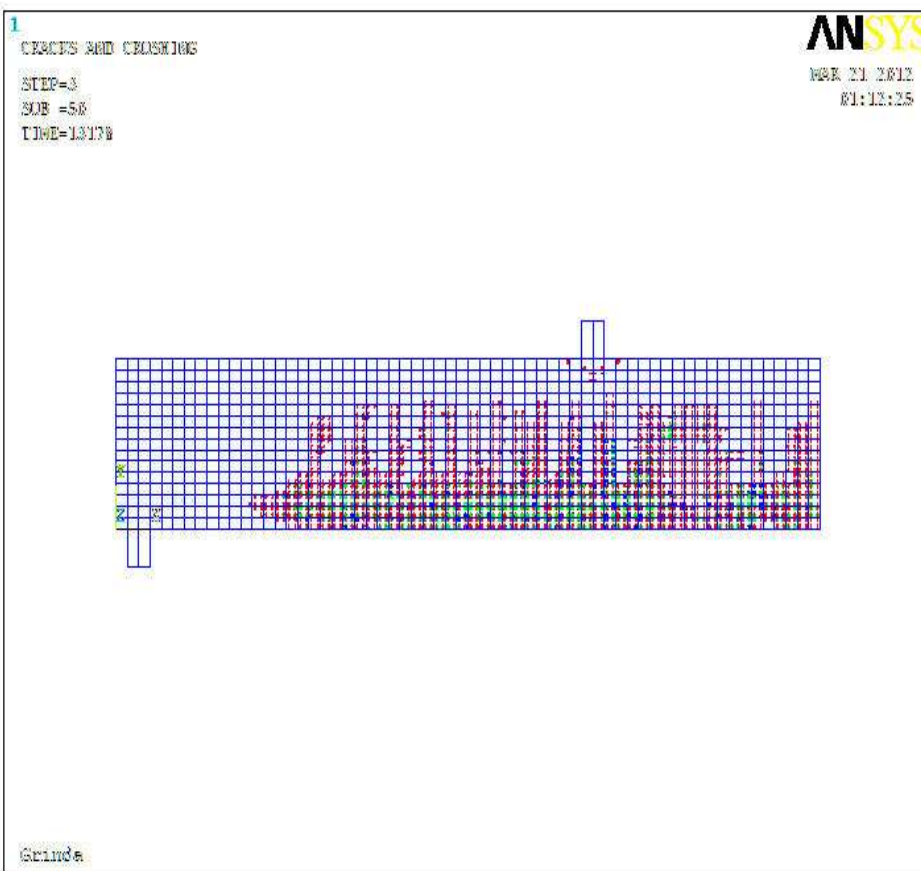
- Fisurile verticale se extind pe inaltimea grinzii si spre reazeme
- Apar fisuri diagonale



GRINDA DE BETON ARMAT(continuare)

Initierea curgerii in armatura

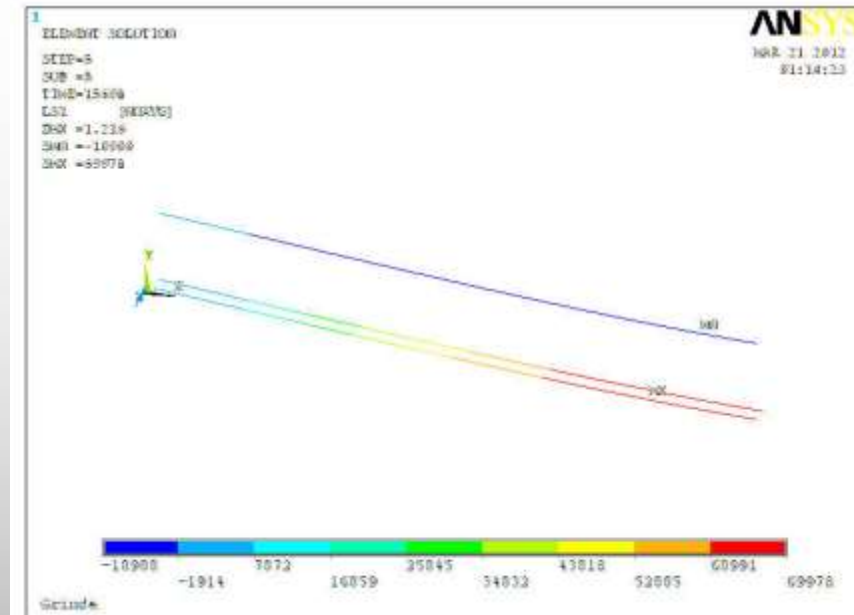
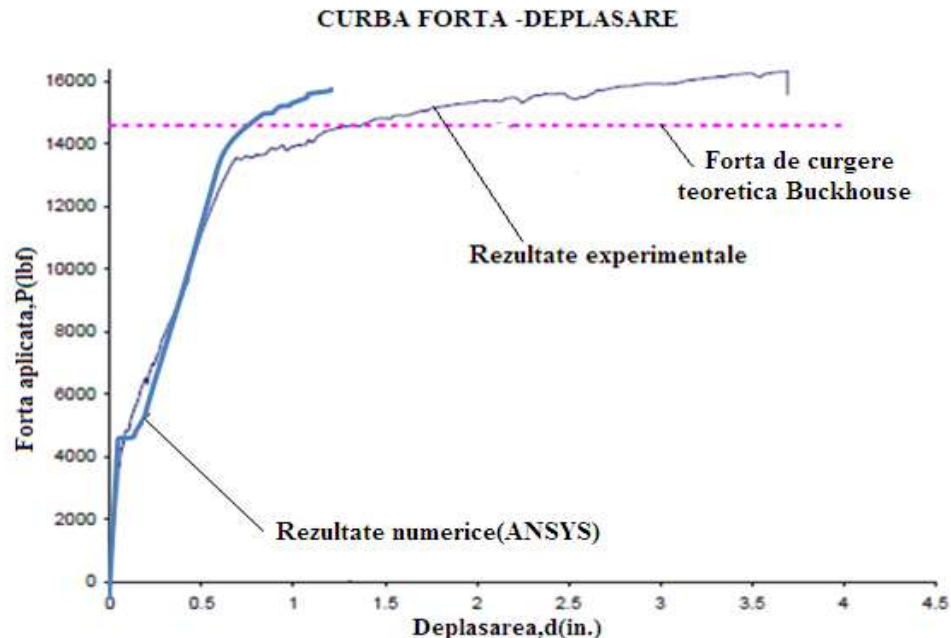
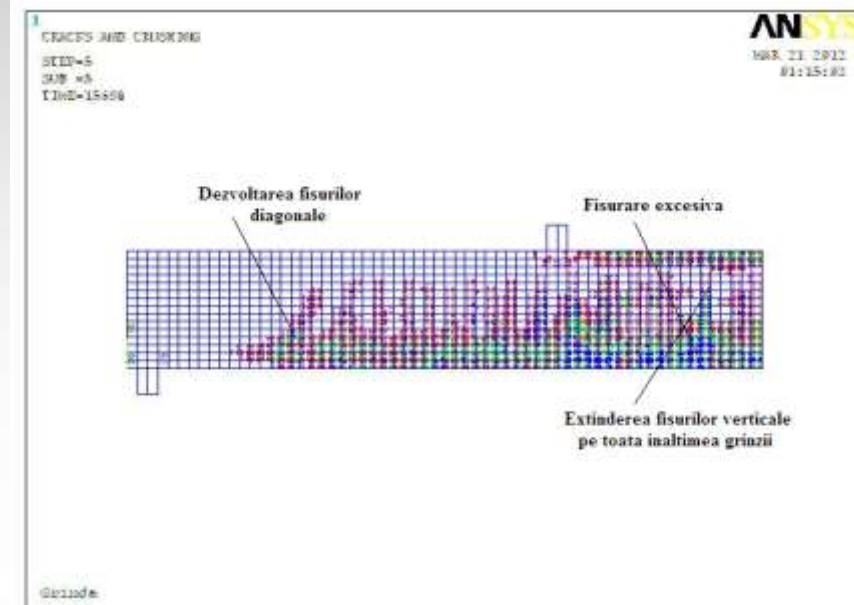
- Curgerea in armatura apare la o valoare a incarcarii $P_y=59$ kN
- Testele experimentale au furnizat o valoare a fortei de curgere $P_y=60$ kN
- Forta de curgere calculata simplificat $P_y=62$ kN



GRINDA DE BETON ARMAT(continuare)

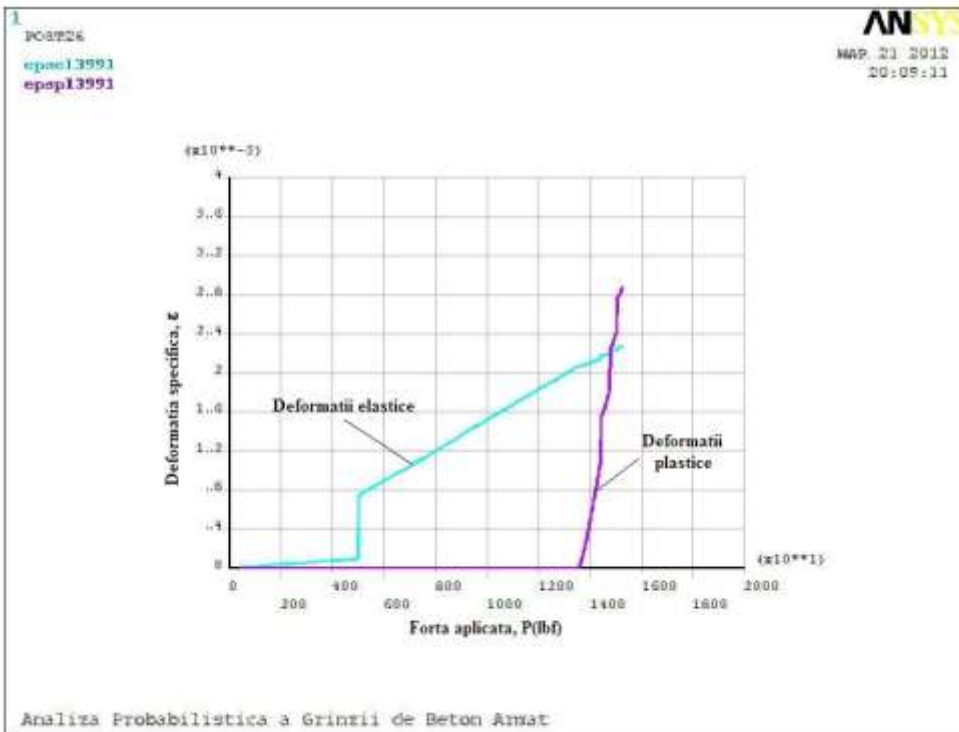
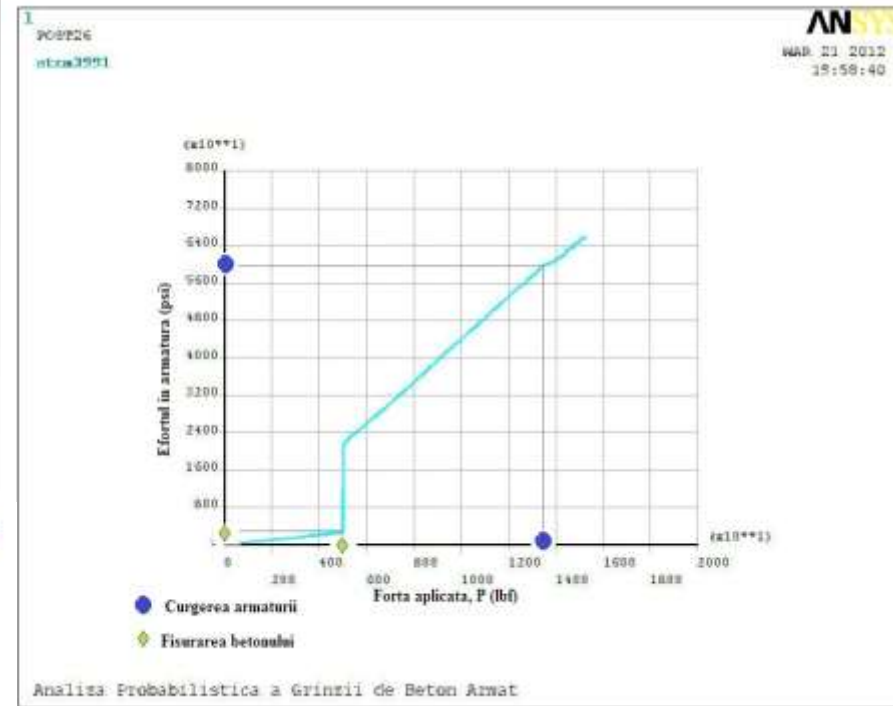
Limita de rupere a grinzii

- Ruperea se produce la o valoare a fortei $P_u=70$ kN
- Forta ultima rezultata din testele experimentale este $P_u=73$ kN
- Efortul in armatura longitudinala corespunzator incarcarii ultime este 483 MPa
- Deplasarea maxima a grinzii : $d=25$ mm



GRINDA DE BETON ARMAT(continuare)

- Dezvoltarea eforturilor in armatura pe masura cresterii fortei aplicate



- Evolutia deformatiile specifice elastice si plastice in armatura

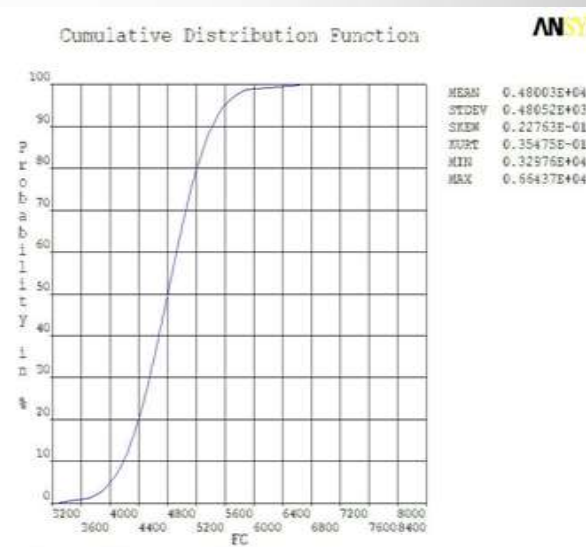
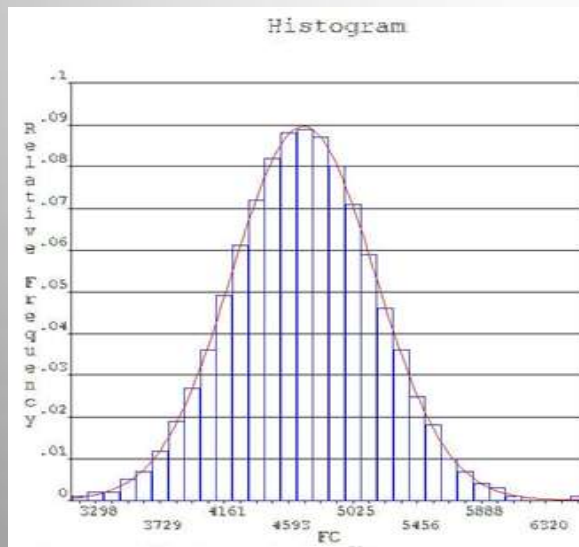
GRINDA DE BETON ARMAT(continuare)

ANALIZA PROBABILISTICA

- S-a considerat variatia a 5 parametri initiali:

Variabila	Distributie	Valoarea medie MPa	Abaterea standard MPa	Coefficient variatie
fc	Gaussian	33.1	3.31	2.57583
ft	Gaussian	3.31	0.3972	2.57583
Ea	Gaussian	199900	7996	2.57583
Eb	Gaussian	27230	2178.4	2.57583
fy	Gaussian	413.70	20.685	2.57583

- Rezultate evidentiate privind doi parametri definitorii: - variatia fortei la care apare curgerea in armatura
- variatia deplasarii



GRINDA DE BETON ARMAT(continuare)

Număr iterație	Variabile de intrare					Variabile rezultate	
	fc MPa	ft MPa	Ea MPa	Eb MPa	fy MPa	Deplasare mm	Forța curgere kN
1	33.10	3.31	199900.00	27230.00	413.70	-25.00	58.74
2	24.57	2.46	199900.00	27230.00	413.70	-20.41	58.54
3	41.62	4.16	199900.00	27230.00	413.70	-26.33	58.79
4	33.10	3.31	199900.00	27230.00	413.70	-28.49	58.37
5	33.10	3.31	199900.00	27230.00	413.70	-24.84	58.74
6	33.10	3.31	179303.67	27230.00	413.70	-28.24	58.74
7	33.10	3.31	220496.33	27230.00	413.70	-25.79	58.62
8	33.10	3.31	199900.00	21618.81	413.70	-28.37	58.49
9	33.10	3.31	199900.00	32841.19	413.70	-28.05	59.11
10	33.10	3.31	199900.00	27230.00	360.42	-23.04	51.37
11	33.10	3.31	199900.00	27230.00	466.98	-28.44	66.05
12	28.83	2.88	189601.83	24424.41	440.34	-26.82	62.15
13	37.36	3.74	189601.83	24424.41	387.06	-27.22	54.92
14	28.83	2.88	189601.83	24424.41	387.06	-23.03	55.23
15	37.36	3.74	189601.83	24424.41	440.34	-27.33	62.42
16	28.83	2.88	210198.17	24424.41	387.06	-25.17	55.12
17	37.36	3.74	210198.17	24424.41	440.34	-27.77	62.56
18	28.83	2.88	210198.17	24424.41	440.34	-27.34	62.93
19	37.36	3.74	210198.17	24424.41	387.06	-23.55	55.86
20	28.83	2.88	189601.83	30035.59	387.06	-22.19	55.12
21	37.36	3.74	189601.83	30035.59	440.34	-26.84	62.95
22	28.83	2.88	189601.83	30035.59	440.34	-24.12	63.06
23	37.36	3.74	189601.83	30035.59	387.06	-21.49	55.62
24	28.83	2.88	210198.17	30035.59	440.34	-27.71	62.15
25	37.36	3.74	210198.17	30035.59	387.06	-24.88	54.69
26	28.83	2.88	210198.17	30035.59	387.06	-21.71	55.00
27	37.36	3.74	210198.17	30035.59	440.34	-24.68	62.42

➤Metoda de analiza:
Central Composite Design

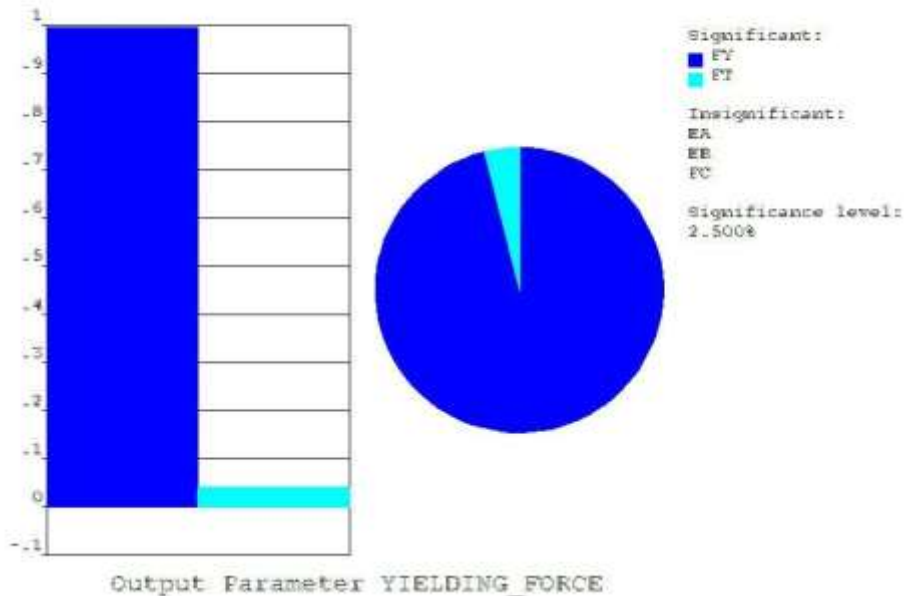
➤27 de experimente pentru
variatia celor 5 parametri

GRINDA DE BETON ARMAT(continuare)

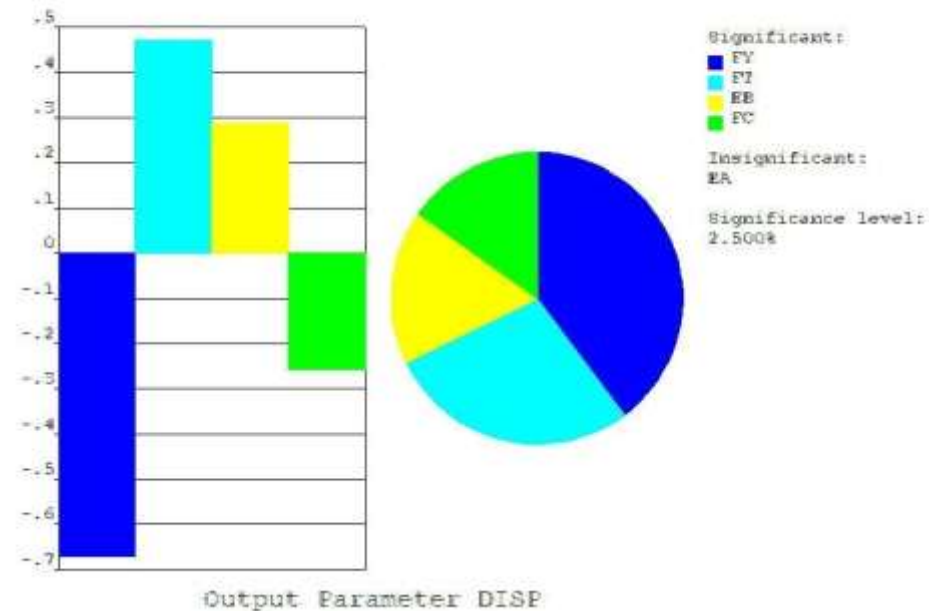
Rezultate-deplasare

- Influenta majora: f_y
- Alti parametri influenti: f_t, E_b, f_c
- E_a nu influenteaza deplasarea

Rank-Order Correlation Sensitivities



Rank-Order Correlation Sensitivities



Rezultate-forța de curgere în armatură

- Influenta majora: f_y
- Alti parametri influenti: f_t
- E_a, E_b și f_c nu influentează forța de curgere

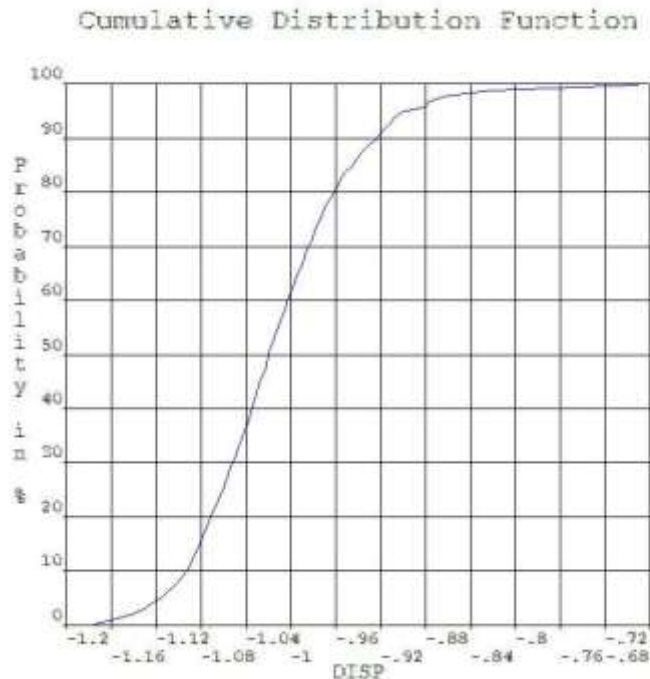
GRINDA DE BETON ARMAT(continuare)

DEPLASAREA

- Histograma si functia de distributie

A rezultat :

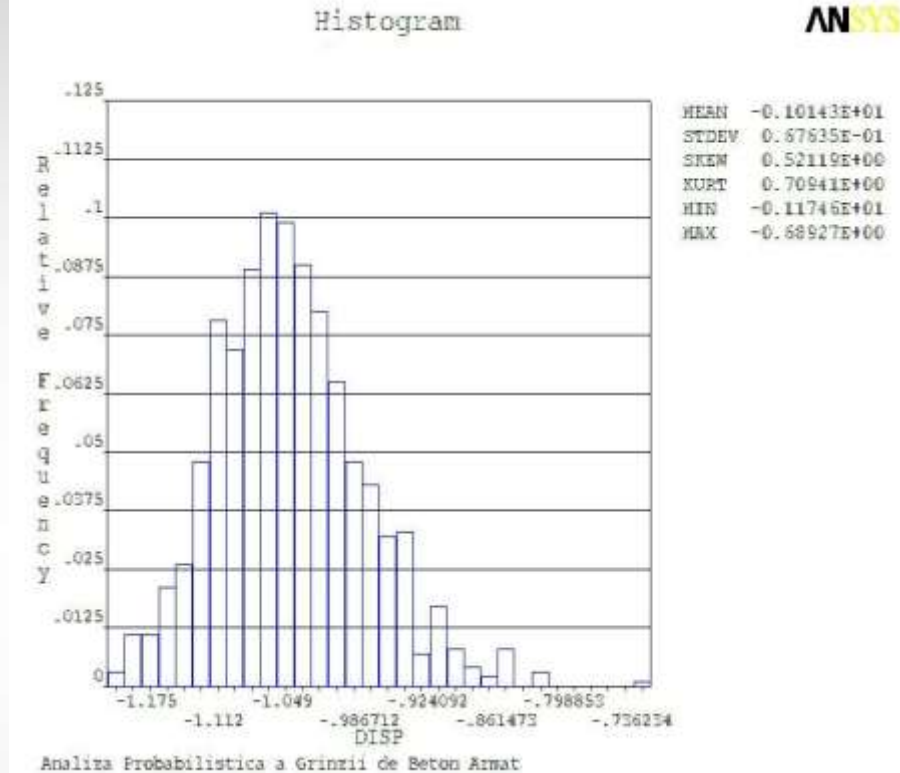
- valoarea medie de 25 mm
- Valoarea minima 17.2 mm
- Valoarea maxima 28 mm



AN **SYS**

MEAN	-0.10143E+01
STDEV	0.67635E-01
SKEW	0.52119E+00
KURT	0.70941E+00
MIN	-0.11746E+01
MAX	-0.68927E+00

Analiza Probabilistica a Grinzii de Beton Armat



GRINDA DE BETON ARMAT(continuare)

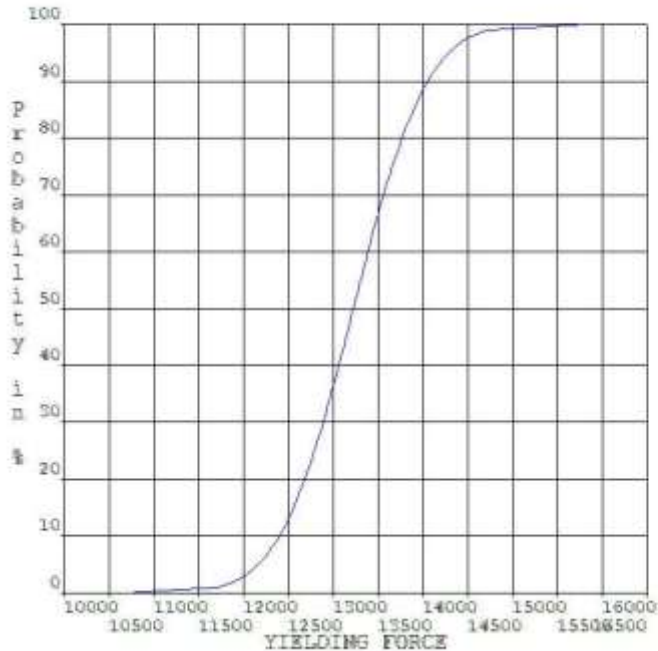
FORTA DE CURGERE IN ARMATURA

- Histograma si functia de distributie

A rezultat :

- valoarea medie de 59 kN
- Valoarea minima 48 kN
- Valoarea maxima 70.5 kN

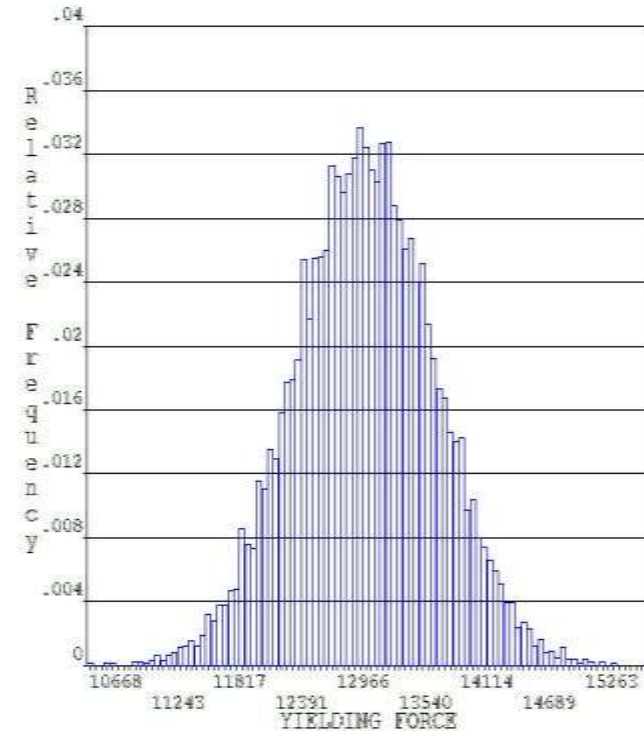
Cumulative Distribution Function



MEAN 0.13221E+05
STDEV 0.64506E+03
SKEW 0.40226E-02
KURT -0.22665E+04
MIN 0.10668E+05
MAX 0.15838E+05

ANSYS

Histogram



MEAN 0.13221E+05
STDEV 0.64506E+03
SKEW 0.40226E-02
KURT -0.22665E+04
MIN 0.10668E+05
MAX 0.15838E+05

ANSYS

GRINDA DE BETON ARMAT(continuare)

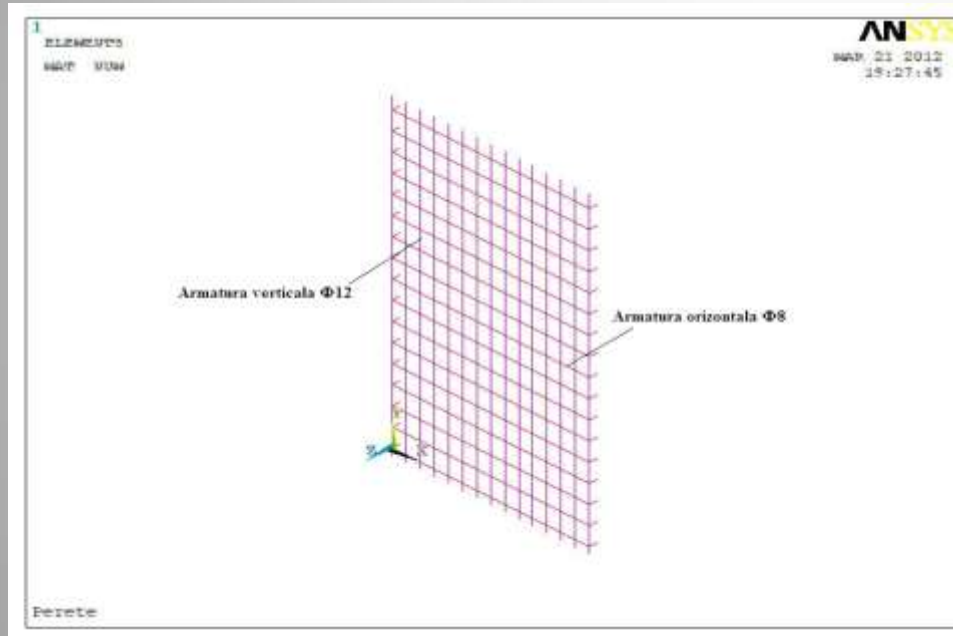
CONCLUZII

- Rezultatele modelului numeric sunt comparabile cu rezultatele testelor experimentale in ceea ce priveste forta la care fisureaza betonul, forta de curgere a armaturii si forta ultima
- Aparitia, dezvoltarea si traseul fisurilor sunt similare intre rezultatele numerice si experimentale
- **Modelul este corect calibrat si elementele folosite pentru modelare, materialele si caracteristicile acestora sunt corespunzatoare pentru a modela comportarea elementului de beton armat.**

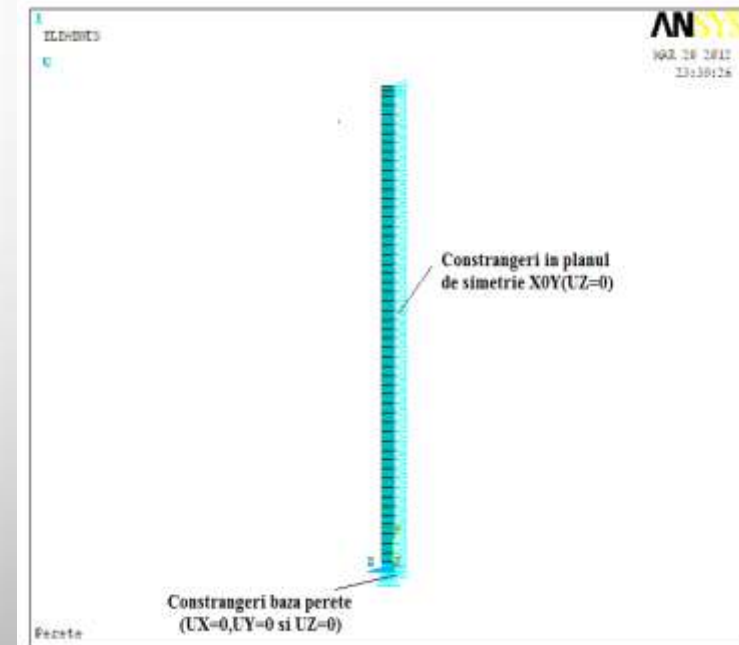
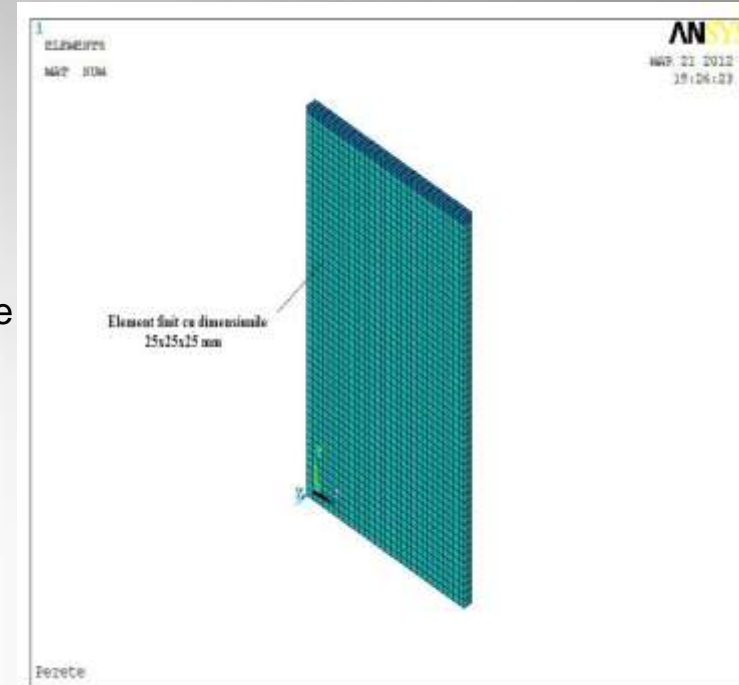
PERETE DE BETON ARMAT

CARACTERISTICI

- Dimensiuni: $L=1500$ mm, $l=300$ mm, $H=2500$ mm
- Armare: $\Phi 12/100$ mm armatura longitudinala si etrieri $\Phi 8/150$ mm
- Datorita simetriei in planul XOY se modeleaza $\frac{1}{2}$ din perete
- Dimensiuni elementefinite: 25 mm



- Pentru a modela simetria se impun conditii in deplasari: $U_Z=0$
- Conditii impuse la baza peretelui: $U_X=0$, $U_Y=0$ si $U_Z=0$
- Incarcare: forta orizontala aplica la partea superioara



PERETE DE BETON ARMAT(continuare)

REZULTATE

Se vor evidenta 3 aspecte:

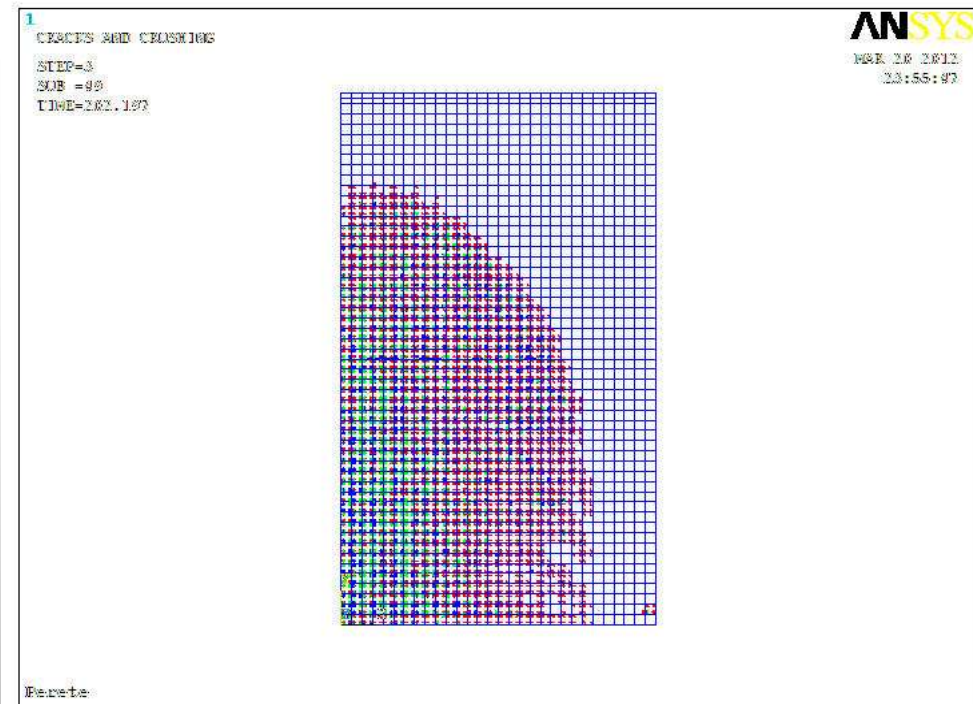
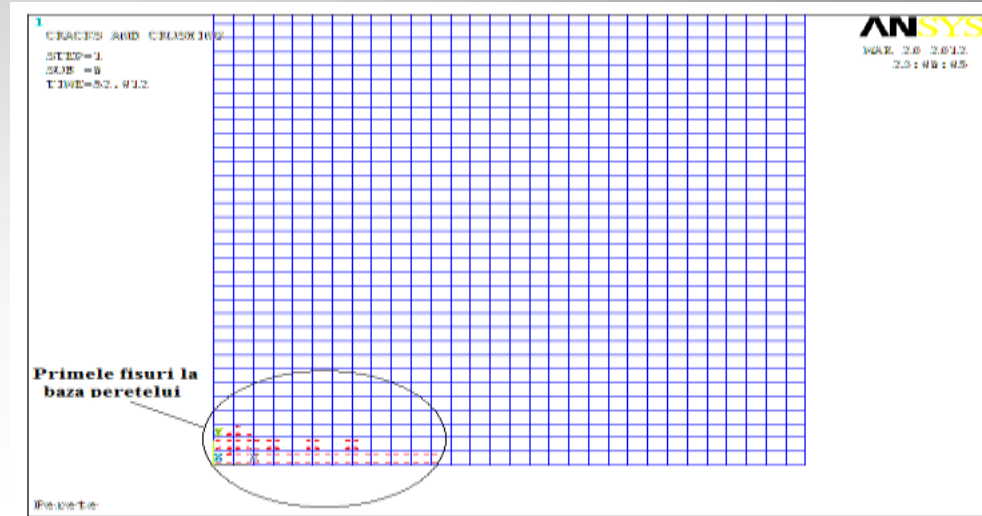
- Fisurarea initiala a peretelui
- Initierea curgerii in armatura
- Limita de rupere

Fisurarea initiala a peretelui

- Fisuri la baza peretelui in zona intinsa a acestuia la o valoare $P=50$ kN
- Forta de fisurare calculata manual: $P=72$ kN

Stadiu intermediar fisuri

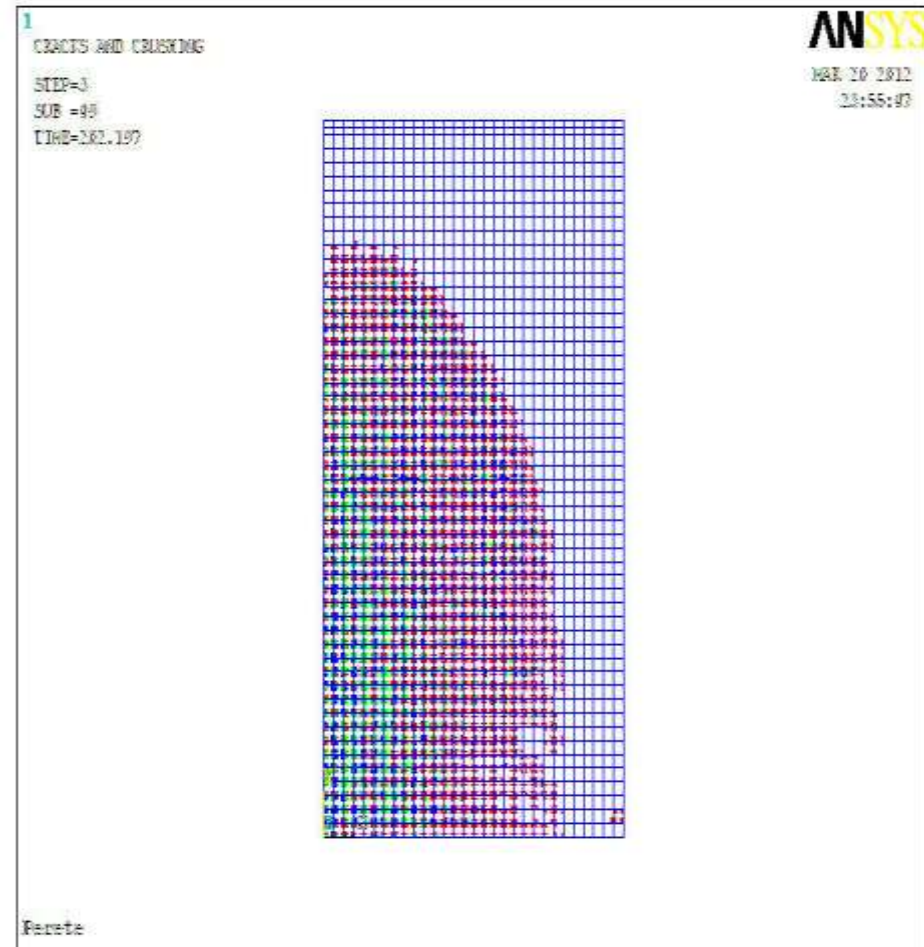
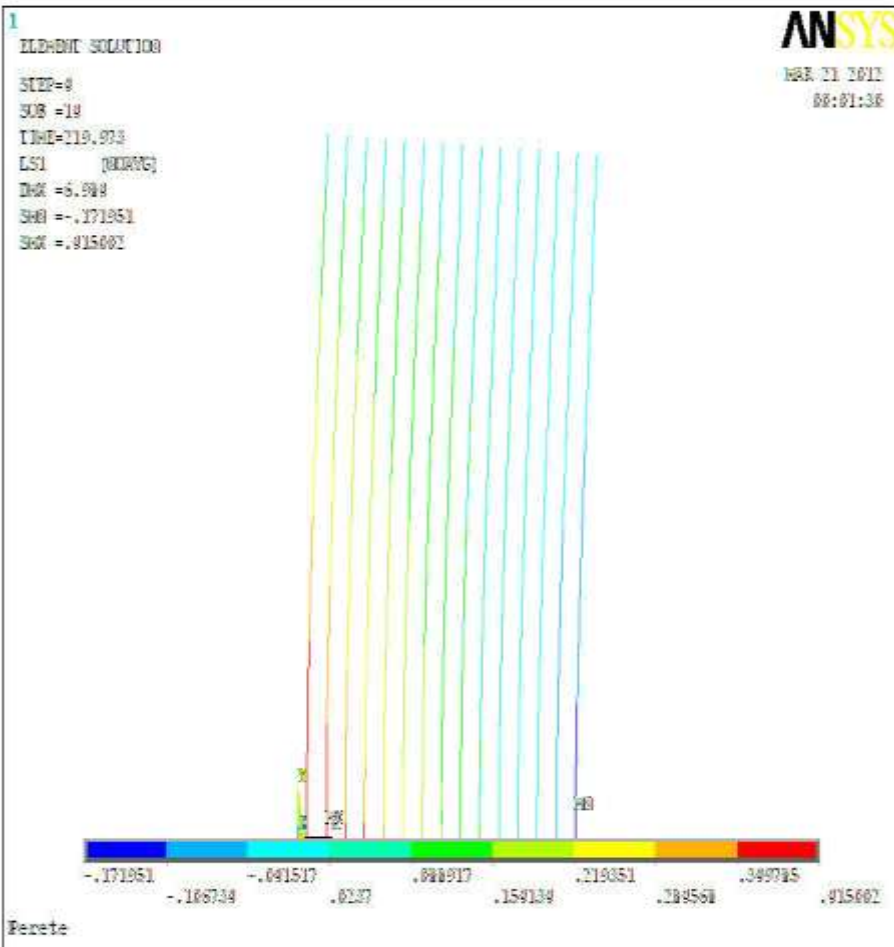
- Fisurile se dezvolta pe inaltimea peretelui
- Apar fisuri in zona comprimata



PERETE DE BETON ARMAT(continuare)

Initierea curgerii in armatura

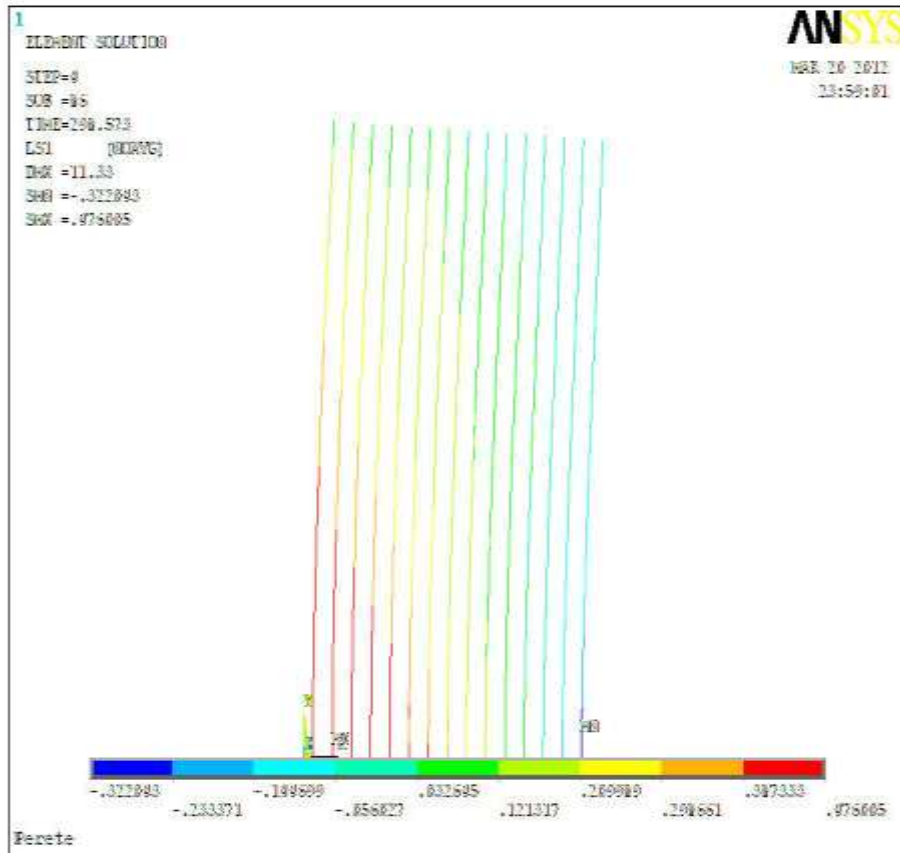
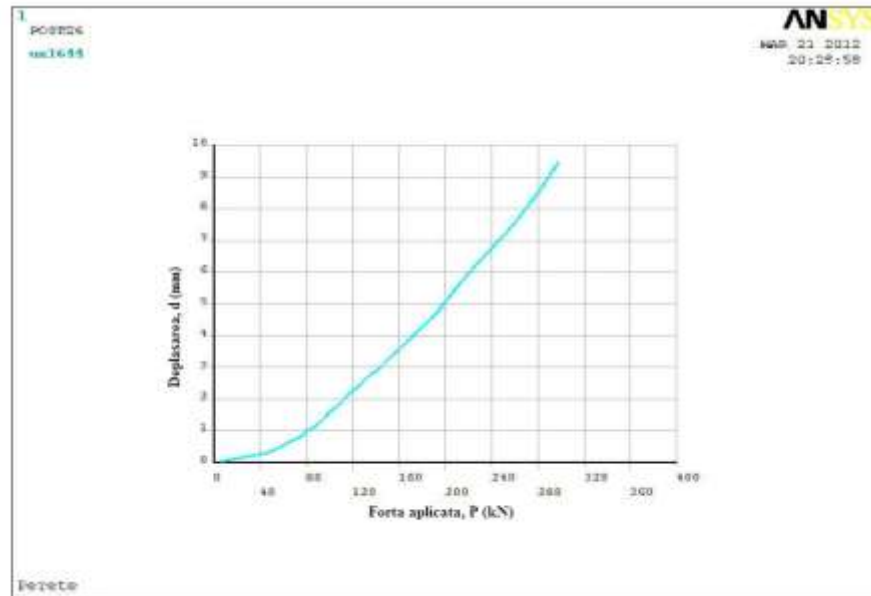
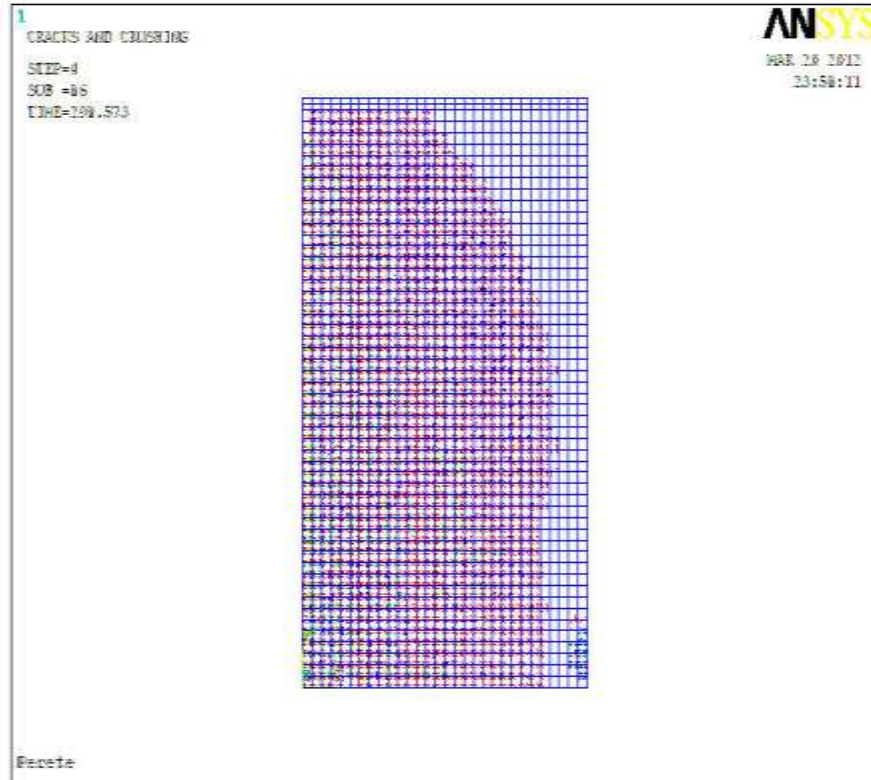
- Curgerea in armatura apare la o valoare a incarcarii $P_y=220$ kN
- Forta de curgere calculata simplificat $P_y=210$ kN



PERETE DE BETON ARMAT

Limita de rupere a peretelui

- Ruperea se produce la o valoare a fortei $P_u=299$ kN
- Efortul in armatura longitudinala corespunzator incarcarii ultime este 476 MPa
- Deplasarea maxima a peretelui : $d=10$ mm



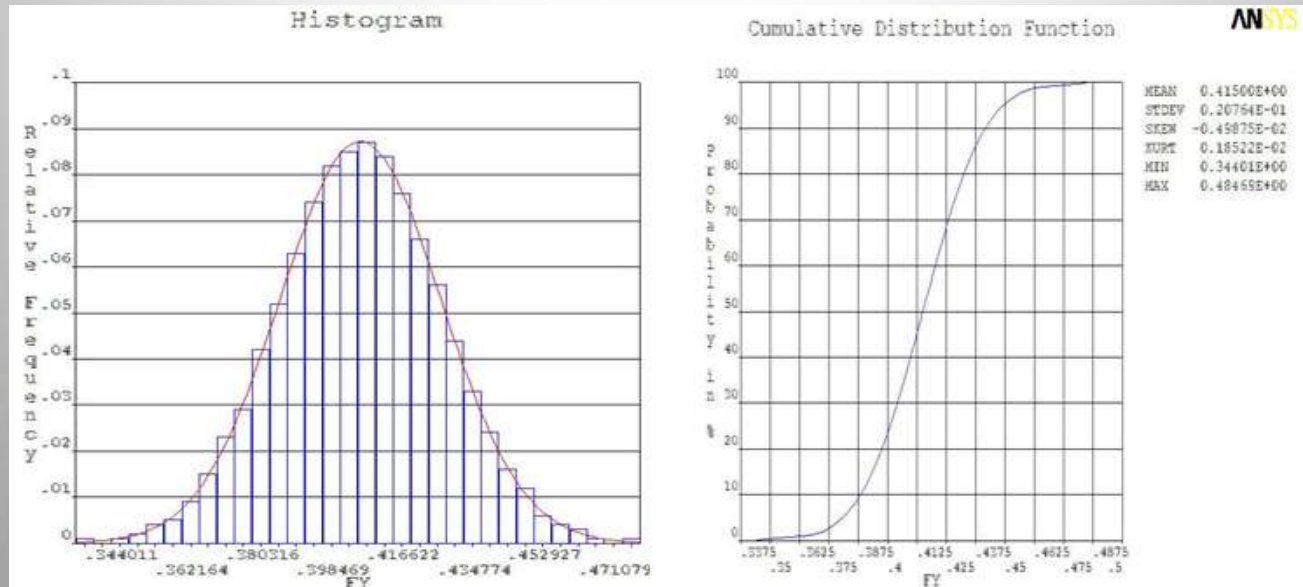
PERETE DE BETON ARMAT(continuare)

ANALIZA PROBABILISTICA

➤ S-a considerat variatia a 5 parametri initiali:

Variabila	Distributie	Valoarea medie MPa	Abaterea standard MPa	Coefficient variatie
fc	Gaussian	33	3.3	2.57583
ft	Gaussian	3.3	0.396	2.57583
Ea	Gaussian	200670	8026.8	2.57583
Eb	Gaussian	27220	2177.6	2.57583
fy	Gaussian	415.00	20.75	2.57583

➤ Rezultate evidentiate in urma analizei: - variatia fortei la care apare curgerea in armatura
- variatia deplasarii



PERETE DE BETON ARMAT(continuare)

Numar iteratie	Variabile de intrare					Variabile rezultate	
	fc MPa	ft MPa	Ea MPa	Eb MPa	fy MPa	Deplasare mm	Fora curgere kN
1	33.000	3.300	200670.00	27220.00	415.000	7.09	219.97
2	33.000	3.300	179994.33	27220.00	415.000	6.84	224.34
3	33.000	3.300	221345.67	27220.00	415.000	6.84	215.61
4	33.000	3.300	200670.00	21610.87	415.000	8.59	210.15
5	33.000	3.300	200670.00	32829.13	415.000	6.04	224.34
6	24.500	3.300	200670.00	27220.00	415.000	4.90	214.56
7	41.500	3.300	200670.00	27220.00	415.000	7.64	218.60
8	33.000	2.280	200670.00	27220.00	415.000	6.53	219.97
9	33.000	4.320	200670.00	27220.00	415.000	6.45	222.16
10	33.000	3.300	200670.00	27220.00	361.552	6.66	191.74
11	33.000	3.300	200670.00	27220.00	468.448	6.89	248.16
12	28.750	2.790	190332.17	24415.44	441.724	8.62	228.46
13	28.750	2.790	211007.83	24415.44	388.276	6.95	200.07
14	28.750	2.790	190332.17	30024.56	388.276	6.38	206.17
15	28.750	2.790	211007.83	30024.56	441.724	5.55	234.22
16	37.250	2.790	190332.17	24415.44	388.276	7.88	205.26
17	37.250	2.790	211007.83	24415.44	441.724	8.90	225.39
18	37.250	2.790	190332.17	30024.56	441.724	6.91	238.20
19	37.250	2.790	211007.83	30024.56	388.276	6.78	209.37
20	28.750	3.810	190332.17	24415.44	388.276	6.67	205.16
21	28.750	3.810	211007.83	24415.44	441.724	6.91	2.38
22	28.750	3.810	190332.17	30024.56	441.724	6.78	234.22
23	28.750	3.810	211007.83	30024.56	388.276	5.47	209.23
24	37.250	3.810	190332.17	24415.44	441.724	8.74	233.54
25	37.250	3.810	211007.83	24415.44	388.276	7.26	198.05
26	37.250	3.810	190332.17	30024.56	388.276	6.97	209.37
27	37.250	3.810	211007.83	30024.56	441.724	7.00	240.53

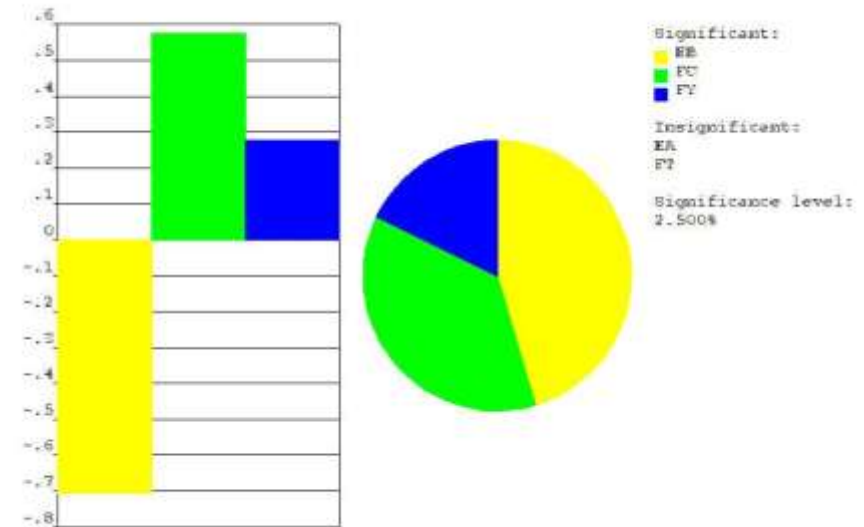
- Metoda de analiza:
Central Composite Design
- **27 de experimente**
pentru variatia celor 5
parametri

PERETE DE BETON ARMAT(continuare)

Rezultate-deplasare

- Influenta majora: Eb
- Alti parametri influenti: fc, fy
- Ea si ft nu influenteaza deplasarea

Rank-Order Correlation Sensitivities



ANSYS

Output Parameter DISP

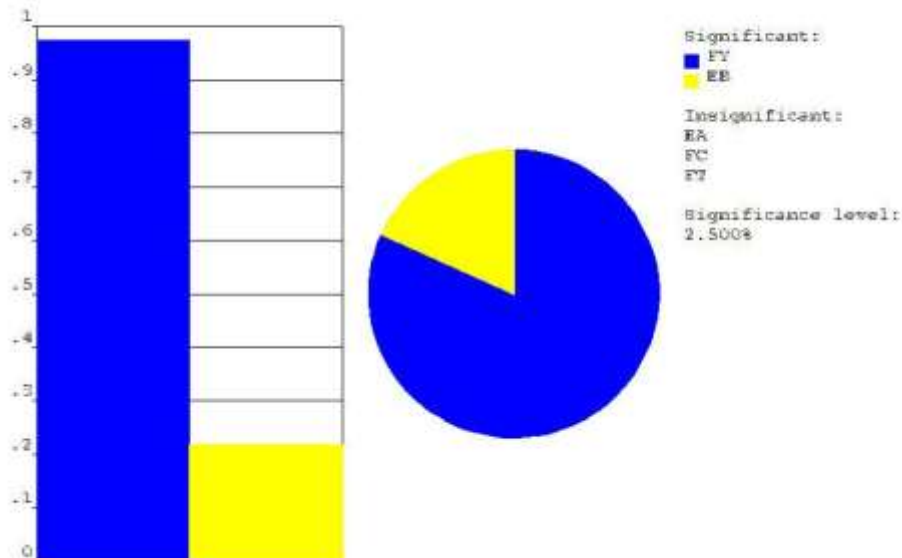
Perete

ANSYS

Rezultate-forța de curgere în armatura

- Influenta majora: fy
- Alti parametri influenti: Eb
- Ea, ft si fc nu influenteaza forța de curgere

Rank-Order Correlation Sensitivities



PERETE DE BETON ARMAT(continuare)

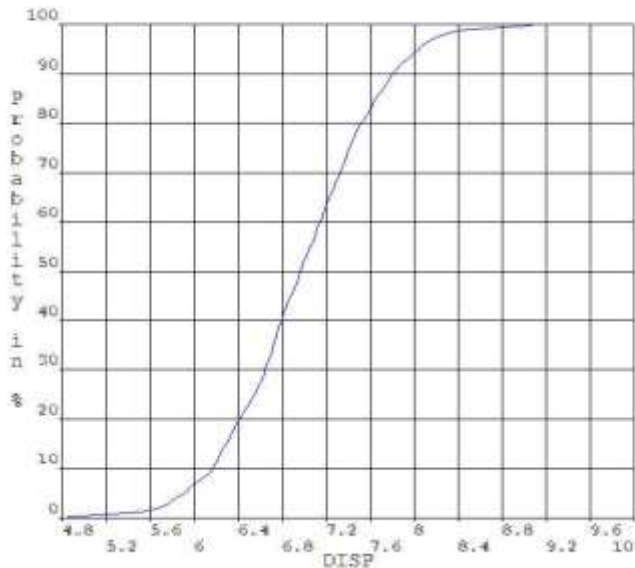
DEPLASAREA

- Histograma si functia de distributie

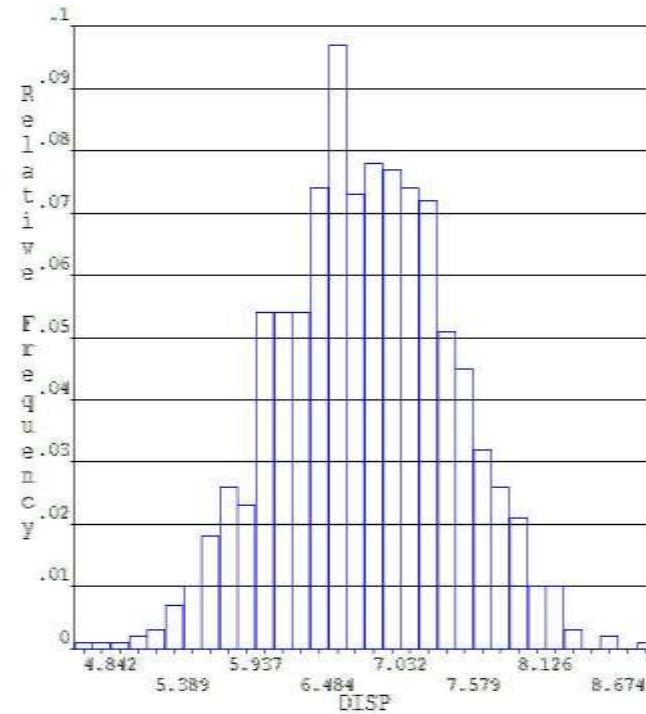
A rezultat :

- valoarea medie de 6.97 mm
- Valoarea minima 4.84 mm
- Valoarea maxima 9.08 mm

Cumulative Distribution Function



Histogram



Statistic	Value
MEAN	0.69716E+01
STDEV	0.64409E+00
SKEW	-0.17136E-01
KURT	-0.12248E+00
MIN	0.48420E+01
MAX	0.90843E+01

Perete

Perete

PERETE DE BETON ARMAT(continuare)

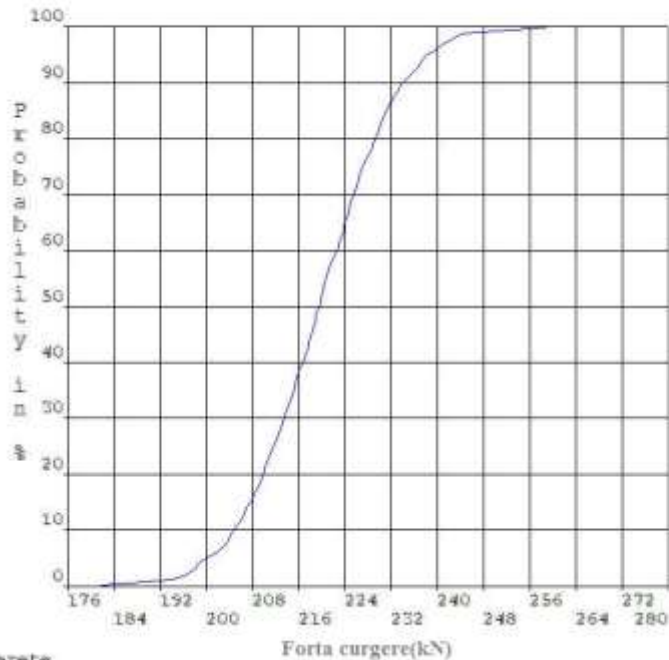
FORTA DE CURGERE IN ARMATURA

- Histograma si functia de distributie

A rezultat :

- valoarea medie de 219.4 kN
- Valoarea minima 181.8 kN
- Valoarea maxima 258.8 kN

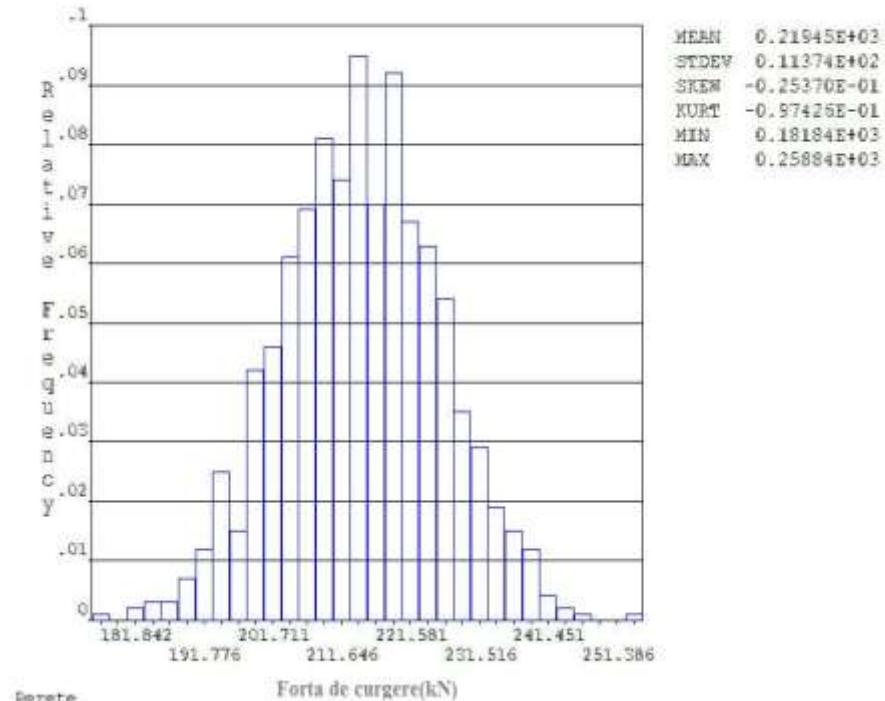
Cumulative Distribution Function



MEAN 0.21945E+03
STDEV 0.11374E+02
SKEN -0.25370E-01
KURT -0.97426E-01
MIN 0.18184E+03
MAX 0.25884E+03

ANSYS

Histogram



MEAN 0.21945E+03
STDEV 0.11374E+02
SKEN -0.25370E-01
KURT -0.97426E-01
MIN 0.18184E+03
MAX 0.25884E+03

Perete

Forta de curgere(kN)

PERETE DE BETON ARMAT(continuare)

CONCLUZII

- Rezultatele modelului numeric privind forta de fisurare a betonului si forta de curgere sunt apropiate de cele calculate cu formule simplificate.
- Comportarea de ansamblu a elementului in ceea ce priveste aparitia, dezvoltarea si traseul fisurilor dar si cedarea sunt conform comportarii reale a peretilor de beton armat
- **Modelarea folosind Metoda Elementului Finit furnizeaza rezultate de incredere pentru determinarea comportarii elementului structural.**

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